

# Solutions Manual

---

# ADVANCED ENGINEERING DESIGN

## Design for Reliability

Engineering problems with solutions covered by the book

This Solutions Manual is intended as working document  
Always check for the Last update  
Last update May 17, 2021

Book: Advanced Engineering Design, edition 2019  
Website: [www.engineering-abc.com](http://www.engineering-abc.com)

Author: Dr.ir. Anton van Beek

Delft University of Technology  
Mechanical Engineering

# CONTENTS

## ADVANCED ENGINEERING DESIGN - LIFETIME PERFORMANCE AND RELIABILITY

---

Chapter 1: Reliability Engineering / Precision Engineering .....	3
Chapter 2: Physics of Failure .....	28
Chapter 3: Fatigue failure prediction and prevention .....	30
Chapter 4: Rolling contact phenomena.....	53
Chapter 5: Friction phenomena.....	63
Chapter 6: Wear mechanisms .....	69
Chapter 7: Material selection.....	77
Chapter 8: Lubricant selection and lubrication management.....	84
Chapter 9: Design of hydrodynamic bearings and sliders .....	88
Chapter 10: Dynamic sealing systems .....	95
Chapter 11: Design of hydrostatic bearings .....	97
Chapter 12: Design of Externally Pressurized Air Bearings .....	104
Chapter 13: Design of flexure mechanisms.....	107
Chapter 14: Machine Design Calculations Reference Guide .....	109
Engineering Equations and Data Tables .....	115

## Chapter 1 - Problems with Solutions

### Problem 1.1: $L_{10}$ service life

Consider a quantity of 10 components that all fail within a year of service. Calculate the  $L_{10}$  service life with 90% reliability and 10% failure probability assuming a normal failure distribution.

Months	1	2	3	4	5	6	7	8	9	10	11	12
Failures	0	0	0	0	0	1	2	4	2	1	0	0

### Problem 1.2: Tolerance field

The diameter of a batch of shafts is normally distributed with 99.7% of the shafts within the tolerance field  $20h9$ , it is between  $20\text{mm } +0 / -52\mu\text{m}$ .

Calculate the size of the tolerance field with 95% probability.



### Problem 1.3: Driving torque interference fit

An interference fit is realized with  $20\text{ H7/r6}$  hole/shaft tolerances. The dimensions of the components are assumed to be normally distributed. The standard deviation is calculated from the assumption that the tolerance interval is a  $\pm 3\sigma$  interval. Linear elastic deformation is to be considered which implies the torque that can be transmitted is proportional to the diametrical interference  $\delta$ .



The torque that can be transmitted, based on the mean value of the diametrical interference, is  $T_{50}$  [Nm]. It is the torque with 50% failure probability. The torque that can be transmitted with 1% failure probability is denoted as  $T_1$ .

The variation of performance, relative to the mean, is a measure of reliability. The coefficient of variation is defined as  $CV' = \text{deviation} / \text{mean}$ . Calculate  $CV' = (T_{50} - T_1) / T_{50}$ .

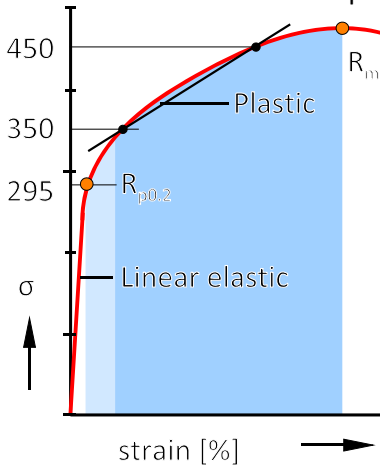
### Problem 1.4: Driving torque tapered shaft hubs

The torque  $T$  that can be transmitted by a tapered shaft-hub connection is proportional to the clamping force, i.e. the bolt preload  $F_i$ . The preload  $F_i$  is proportional to  $M_A / \mu$  where  $M_A$  is the tightening torque  $\mu$  the coefficient of friction in the screw assembly. The coefficient of friction  $\mu$  is managed by using a proper thread lubricant and varies between 0.12 and 0.16. Calculate the coefficient of variation  $CV' = (T_{50} - T_{\min}) / T_{50}$  where  $T_{\min}$  is the least torque that can be transmitted by the shaft-hub connection.



### Problem 1.5: Interference fit with hollow shaft

A gear is to be press fitted over a hollow shaft of 20 mm diameter. Consider the interference  $\delta = 0.15 \pm 0.05$  mm. The strain is that much that the hollow shaft will deform plastically.



The tensile stress in the plastic regime varies much less with the strain than in the elastic regime.

The tensile stress in the plastic regime of the steel shaft is approximated by linear interpolation between  $\sigma(\epsilon=0.005) = 350$  MPa and  $\sigma(\epsilon=0.01) = 450$  MPa. Calculate the coefficient of variation  $CV' = (T_{50} - T_{\min}) / T_{50}$  where  $T_{\min}$  is the driving torque that can be guaranteed for all connections.

### Problem 1.6: Estimation of service interval

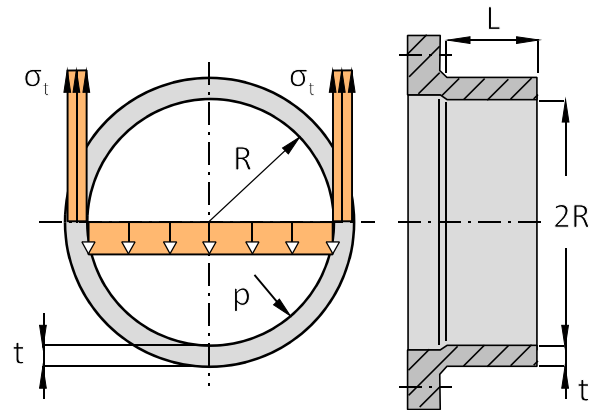
The connection between a thin walled aluminium hub and a steel shaft is realised by means of a pressure fit. The deformation of the hub when mounted is in the plastic regime. The contact pressure between the clamped parts is proportional to the tensile stress that is created in the aluminium hub during assembly.

Mathematics of uncorrelated variables

$$\sigma_{x \pm y} = \sqrt{\sigma_x^2 + \sigma_y^2}$$

$$\sigma_{aX+b} = \sqrt{a^2 \sigma_x^2} = a \sigma_x$$

$$\sigma_{xy} = \sqrt{\sigma_x^2 \sigma_y^2 + \sigma_x^2 \mu_y^2 + \sigma_y^2 \mu_x^2}$$



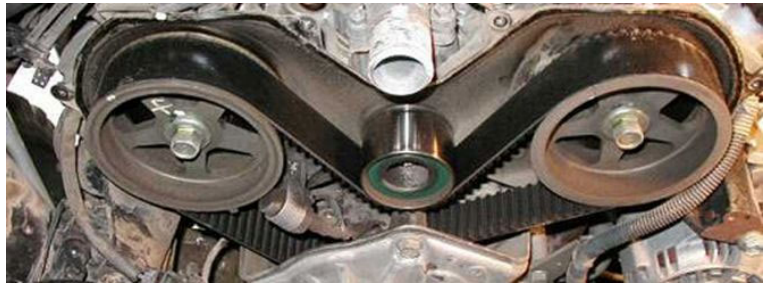
Consider a batch of shafts and hubs that are assembled. The tensile stress in the hub depends on the size of the interference and varies between  $200 \text{ MPa} < \sigma_t < 250 \text{ MPa}$ . The coefficient of friction varies between  $0.15 < \mu_f < 0.25$ .

The torque  $T$  that can be transmitted by a pressure fit is related by  $T = a \cdot \mu_f \cdot \sigma_t$  where "a" is a constant,  $\mu_f$  the coefficient of friction and  $\sigma_t$  the tensile stress.

- Calculate the coefficient of variation  $CV'$  of the torque where  $CV'$  is based on the worst case (extreme) torque that can be transmitted (deterministic approach).
- Calculate the coefficient of variation  $CV'$  of the torque when the variation is based on a  $\pm 3\sigma$  normal distribution of the tensile stress and a  $\pm 3\sigma$  normal distribution of the coefficient of friction (probabilistic approach).

### Problem 1.7: Estimation of service interval

From a series of experiments it is found that the component life is  $\mu = 150 \cdot 10^3$  km and  $\sigma = 20 \cdot 10^3$  km. A component reliability of 90% is specified with  $L_{10}$ , of 99% with  $L_1$ . Calculate the value  $L_{10}$  and  $L_1$  and the ratio  $a_1 = L_1/L_{10}$ .



### Problem 1.8: Conversion of MTBF to Reliability

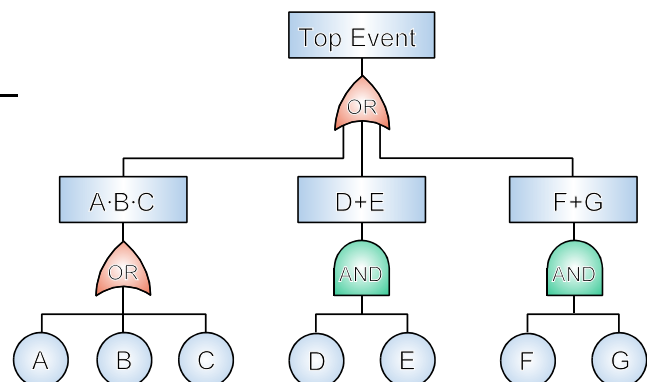
- Estimate the MTBF for  $N=10$  devices that are tested for  $T_{\text{test}}=500$  hours and during the test  $r=2$  failures occur.
- Estimate the probability that any one particular device will be operational at the time equal to the MTBF?
- Estimate the probability that the component will work for 50% of the MTBF
- Estimate the percentage of the MTBF where  $R(t)=0.95$ .



### Problem 1.9: Fault Tree Analysis

Consider the fault tree with the component reliability given in the table below and calculate the failure probability  $F(t)$  of the system for a service life  $t$ .

	A	B	C	D	E	F	G
$R(t)$	0.9	0.85	0.9	0.7	0.95	0.8	0.99



**Problem 1.10: Bearing reliability, deep groove ball bearing**

Calculate the operating reliability  $R(t=1000\text{hr})$  of a deep groove ball bearing. The calculated  $L_{10}$  life expectancy of the ball bearing is  $L_{10} = 500 \cdot 10^6$  rev. The rotational speed is 4000 rpm.

Hint: The life expectancy of the ball bearings is related to the  $L_{10}$  basic rating life according (eq. 1.18, page 19).

**Problem 1.11: Reliability factor for Fatigue strength**

A power supply is cooled by 3 fans. The correct functioning of at least one of the three fans is required to maintain sufficient cooling. The operating reliability of the system needs to be 99% for a service life of 10.000 hr,  $R_s(10.000 \text{ hr}) = 0.99$ . The rotational speed is 4000 rpm.

- Estimate the required operating reliability  $R_j$  of the individual fans.
- Calculate the required  $L_{10h}$  of the individual bearings.
- Calculate  $C/P$

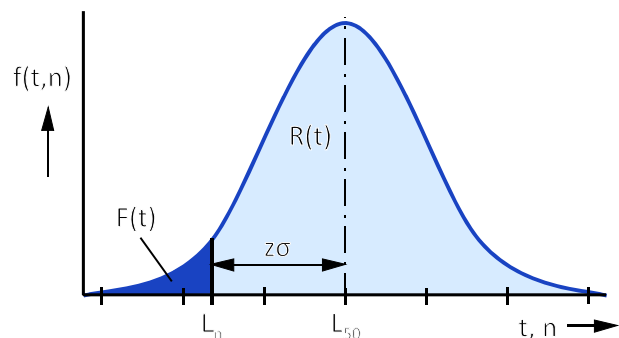
**Problem 1.12: Reliability factor for Fatigue strength**

Data published of the endurance strength are always mean values. In Norton (2000) is reported that the standard deviation of the endurance strength of steels seldom exceeds 8% of their mean. Estimate a correction factor for the endurance strength if a 99% probability is required.

**Problem 1.13: Component reliability**

Calculate the component reliability of a drive shaft (motor shaft) loaded in the High Cycle Fatigue (HCF) regime with  $L_n = 2 \cdot 10^5$  load cycles.

Consider the calculated fatigue life of  $L_{50} = 3 \cdot 10^5$  load cycles and a standard deviation of  $\sigma = 0.2\mu$ .

**Problem 1.14: Stress concentration factor**

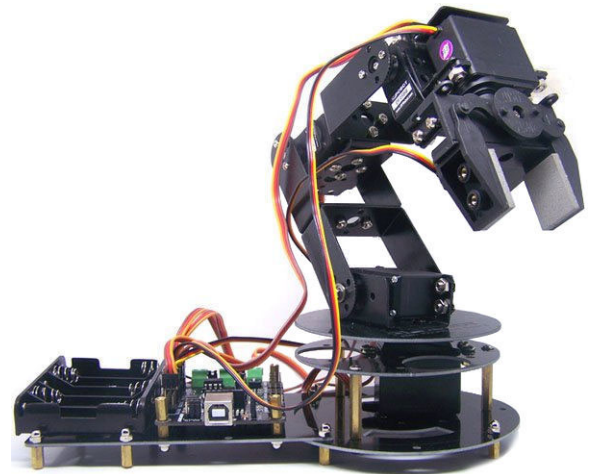
"A chain is only as strong as its weakest link, regardless of the strength of the stronger links". Do you agree and what do you think about the reliability if the failure mode is fatigue?



### Problem 1.15: System reliability

Calculate the failure probability  $F(100 \text{ hr})$  of two critical components of a system connected in series. From product catalogues it is found that the reliability of one component is specified with  $\mu = 150 \text{ hr}$  and  $\sigma = 30.5 \text{ hr}$ , the other component is specified with  $\mu = 120 \text{ hr}$  and  $\sigma = 10.2 \text{ hr}$ .

Hint: first step is to calculate  $R(t)$  of both components.



### Problem 1.16: System reliability

A heavy-duty motorized frame features a quad drive system using two high power DC motors and four drive belts. All four belts are required to maintain optimal control. From field testing it is found that the service life of the belts under heavy duty operating conditions is normally distributed with a mean  $\mu = 200 \text{ hr}$  and a standard deviation of  $\sigma = 0.2\mu$ . Calculate the operating reliability of the set of 4 belts for a service life of 150 hr.



### Problem 1.17: System reliability

There is a rule of thumb that says that the bearing load  $P$  related to the dynamic load rating of the bearing  $C$  is:

Light loaded bearings  $P \leq 0.06C$

Normal loaded bearings  $0.06C < P \leq 0.12C$

Heavily loaded bearings  $P > 0.12C$

Consider a motor drive equipped with two ball bearings. One of the bearings is loaded with  $P=0.1C$ , where  $C$  is the dynamic load rating of bearing type 16004,  $C=7.28 \text{ kN}$ . The motor rotates with  $n=1400 \text{ rpm}$  during 8 hours a day, 5 days a week it is 1920 hr/year. Calculate the life expectancy [years] of this bearing with 1% failure probability.

Hint: First calculated  $L_{10}$  and  $L_{10h}$ . The life expectancy  $L_{na}$  of ball bearings is related to the  $L_{10}$  basic rating life according  $L_{na} = a_1 L_{10}$  (eq.1.18 page 19).



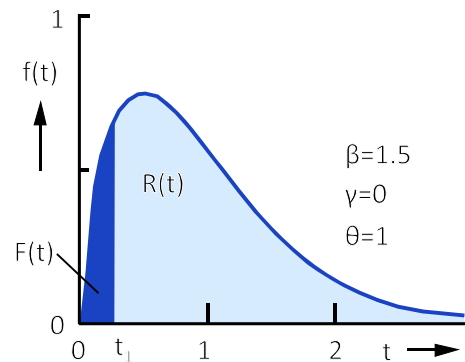


**Problem 1.18: System reliability**

Consider a motor drive equipped with two ball bearings. The life expectancy of the bearings is calculated as  $L_{10h}=12,000$  hr and  $L_{10h}=16,000$  hour respectively. Calculate the system (motor) reliability for a service life of 10,000 hour.

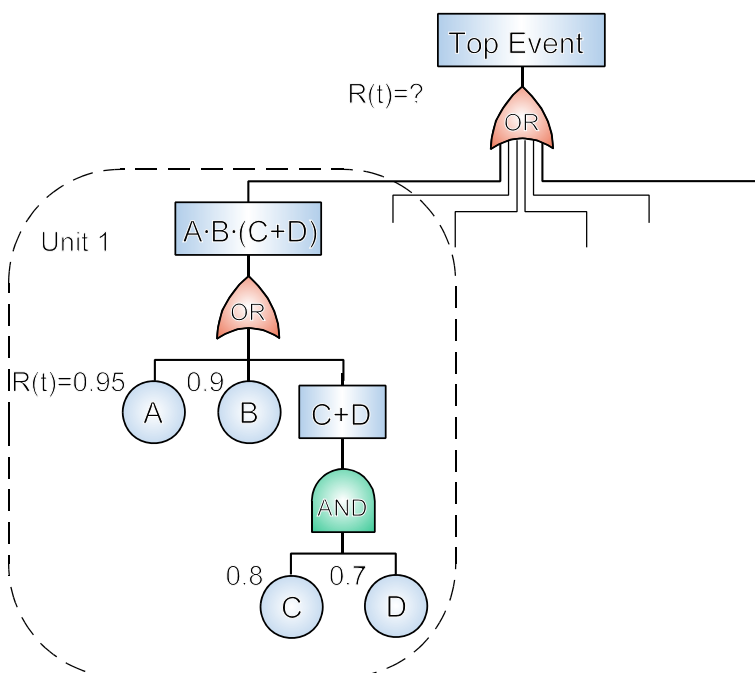
a) The life expectancy  $L_{na}$  of the ball bearings is related to the  $L_{10}$  basic rating life according  $L_{na}=a_1 L_{10}$  (eq.1.18 page 19) where  $a_1$  is the reliability factor derived from a statistical “Weibull distribution”.

b) It is assumed that the failure distribution  $f(t)$  of the bearings is best fitted by a Weibull Failure Distribution function with shape factor  $\beta=1.5$  (Eq. 1.5 page 12).

**Problem 1.19: System reliability**

The operating reliability of a production line needs to be estimated. The production line consists of 6 identical pick and place units with similar operating conditions.

The most critical components of the individual pick and place units are identified using an FMEA procedure. The operating reliability of these components are established and finally presented in a fault tree.



Calculated value of the operating reliability of the production line  $R(t)$ .

Which of the components would you select to improve its operating reliability by 5 percent in order to improve the system reliability?

### Problem 1.20: System reliability

A monitoring unit is applied to register as a function of time unexpected machine standstill which is caused by a specific component in the pick and place units. The service life  $t$  (hr) of this component is derived from the monitored data and listed in the table below.

Service life  $t$  (hr) of the critical component

590	420	520	480	490	510	450	480
-----	-----	-----	-----	-----	-----	-----	-----

Calculate the required maintenance interval in order to replace this component in all pick and place units in time. The reliability of the individual pick and place units should be at least  $R(t)=0.99$ . What is  $t$ ?



### Problem 1.21: True or Untrue?

Key words: Reliability and Failure probability, FMEA, FTA, RCA.

1. Reliability is defined as the probability that a system delivers the required performance during a certain time interval (page 11)
2. The failure probability  $F(t=\mu)$ , for a service life  $t$  equal to the mean value  $\mu$  of the failure distribution is  $F(t=\mu) = 50\%$  (page 13).
3. System reliability decreases with components connected in parallel (page 16).
4. Fault Tree analysis (FTA) is usually conducted before a Failure Mode and Effect Analysis (FMEA) in order to identify the possible failure modes of a system (page 20).
5. FMEA gives good insight in system reliability, the probability that the system performs well during a certain time interval (page 20).
6. Proactive maintenance is performed with the goal to obtain a good balance between maintenance costs and operational losses (page 20)
7. The failure probability of two bearings in a motor drive is calculated by multiplying the failure probability of the individual bearings (page 19).
8. The reliability of two bearings in a motor drive is calculated by multiplying the reliability of the individual bearings (page 19).
9. The system reliability will increase by redundancy, that is obtained by connecting components in parallel (page 16).
10. The failure probability of a power transmission, in which an interference fit is used to transmit a specified minimum value of power, is represented by a two-sided confidence interval (page 15).

**Problem 1.22: Precision**

The quality of two plotters needs to be compared. Both plotters are actuated  $n$  times to an intended position of  $x_{\text{target}} = 100$  mm. The recorded positions can be fitted by normal distributions.

- Plotter 1 shows  $\mu = 101$  mm,  $\sigma = 0.5$  mm
- Plotter 2 shows  $\mu = 98$  mm,  $\sigma = 0.2$  mm

Which of the two plotters is the most precise?

**Problem 1.23: Precision and Accuracy**

The quality of a plotter is established by testing. In this test the pencil of the plotter is actuated 8 times to an intended position of  $x_{\text{target}} = 100$  mm. The realised positions are recorded and listed in the table below. It is assumed that the data can be fitted by a normal distribution.

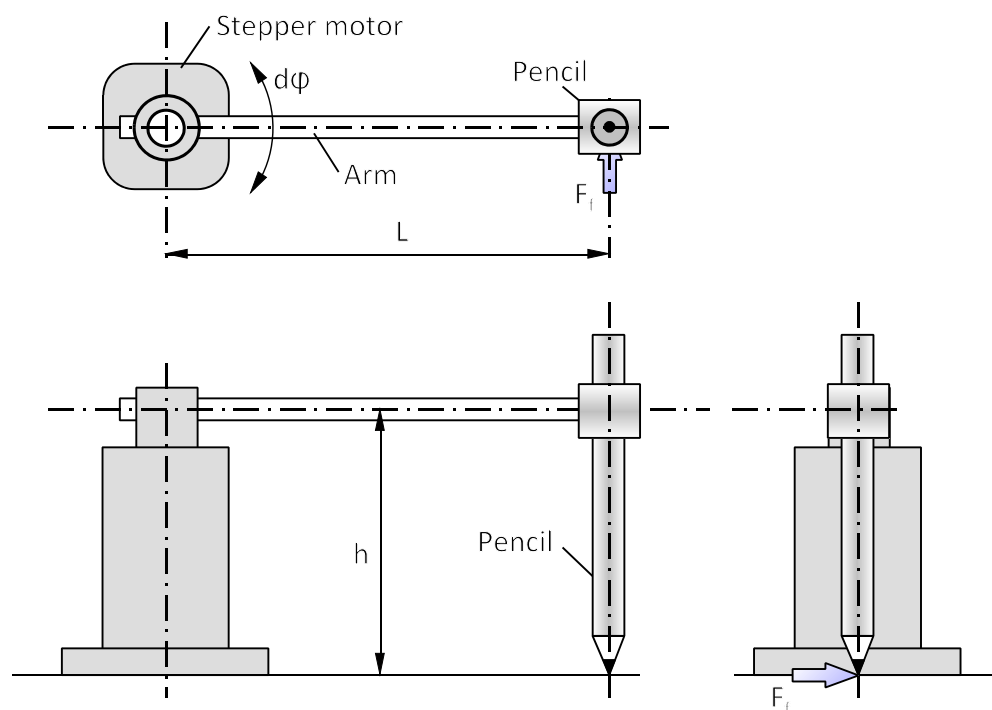
Measured data [mm]    99.2    99.4    99.4    99.5    99.7    99.8    99.8    100.0

- a) Calculate the precision interval  $P$  [ mm ] with 95% probability.
- b) Calculate the accuracy  $A$  of this plotter.
- c) What will the accuracy be when the offset is corrected with  $\delta = x_{\text{target}} - x_{\text{mean}}$ .

**Problem 1.24: Virtual Play**

Consider a simplified model of a robot arm that is actuated by a stepper motor using 200 steps/rotation. The beam length  $L$  measured from the shaft centre of the stepper motor to the centerline of contact between the pencil and the paper is  $L=0.2$  m. The beam has a diameter of  $d=5$  mm and is made of steel with  $E=210$  GPa,  $G=80$  GPa. The distance from the clamping of the pencil to the point of contact with the paper is  $h=0.15$  m. The bending stiffness of the pencil as measured at the tip of the pencil is  $k=1$  N/mm. The friction force between the pencil and the paper is  $F_f = 2$  N.

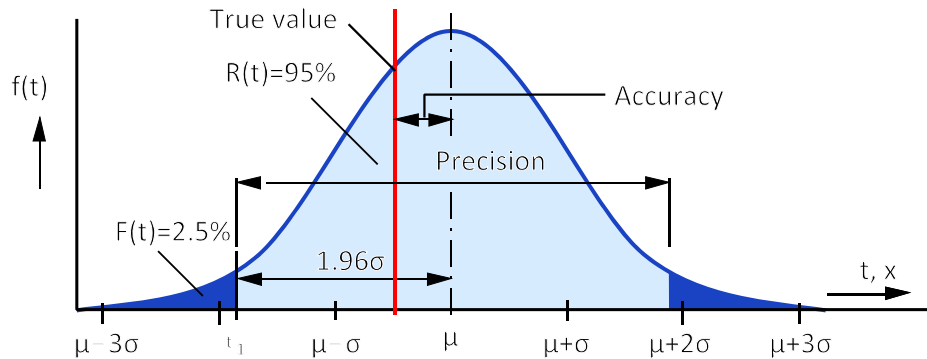
Calculate the ratio  $\delta_b / \delta_p$ , where  $\delta_b$  is the partition of the virtual play as a result of the flexibility of the beam and  $\delta_p$  is the partition of the virtual play as a result of the flexibility of the pencil.



**Problem 1.25: Number of measurements needed to obtain a reliable estimation**

When the measurement of the coefficient of friction is repeated a large variation will be found. Consider the measured values 0.11, 0.12, 0.13, 0.14 and 0.15.

- Calculate the 95% interval over which the coefficient of friction may lie.
- What number of measurements are needed to estimate the mean with 95% reliability within  $\pm 0.01$  accuracy?

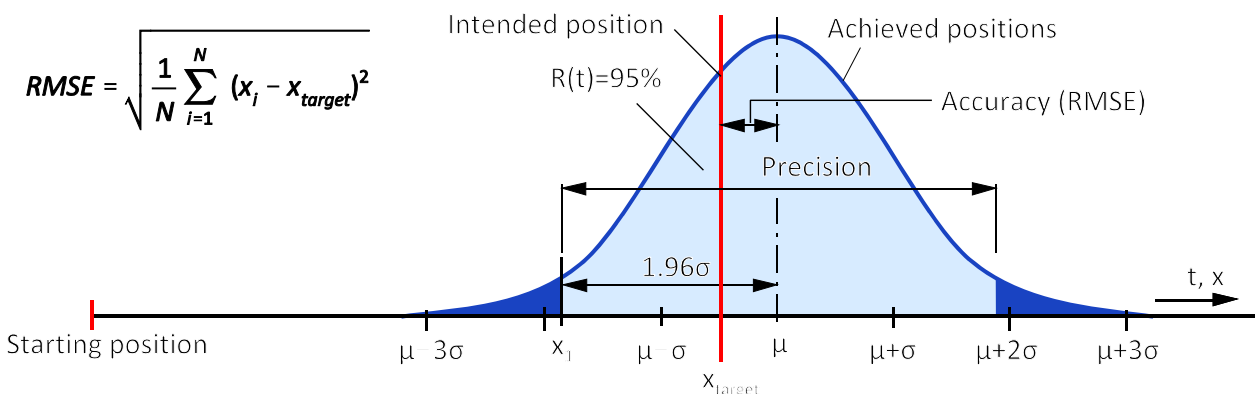

**Problem 1.26: Precision and accuracy**

**Repeatability (precision)** is the error between a number of successive attempts to move the machine to the same position. Repeatability can be represented by the interval which contains N% of the measured positions.

**Accuracy** is the difference between the intended position and the mean of the measured positions. The root-mean-square deviation (RMSD) or root-mean-square error (RMSE) is a frequently used measure for accuracy. The accuracy can be improved by adjusting the offset. The precision remains the same.



**Resolution** is the smallest possible movement of a system when actuated. Also known as the step size. The resolution of an instrument is the smallest increment that the gage displays.

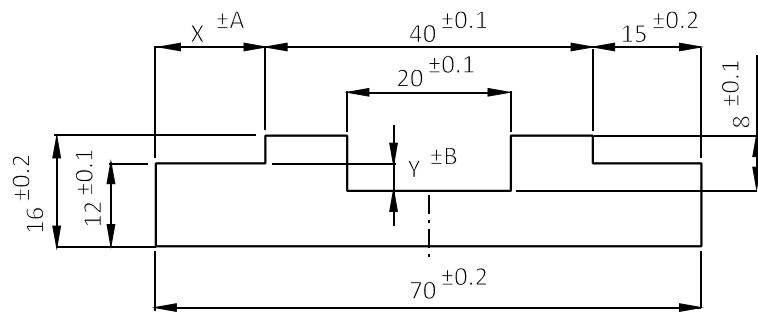


$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - x_{target})^2}$$

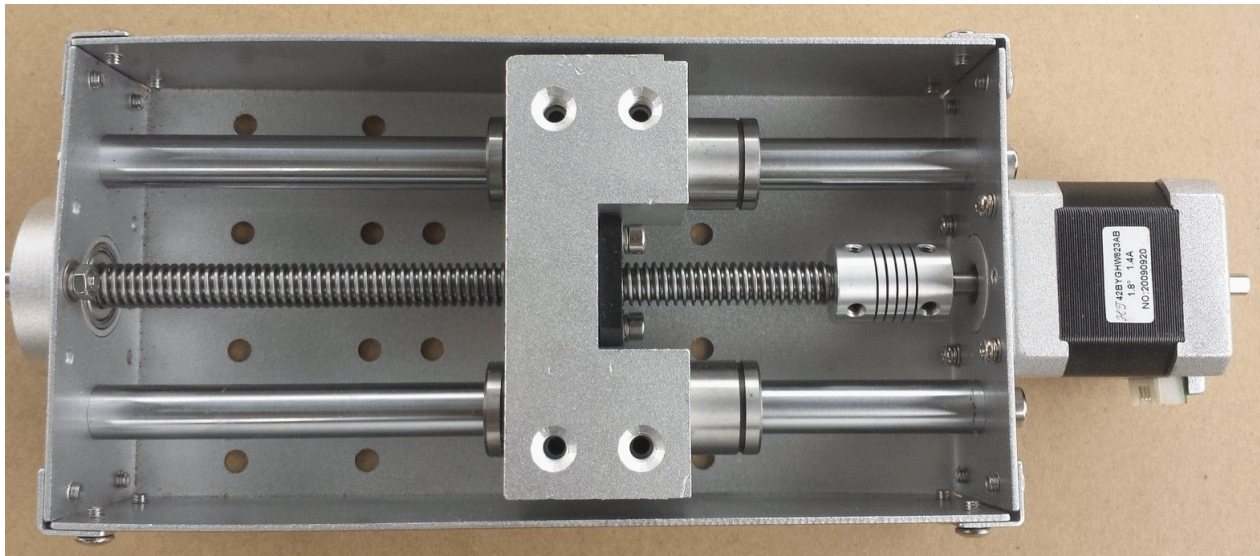
Consider an intended position of  $x_{target} = 40$  mm and a range of measured  $x$ -data: 40.12, 40.07, 39.98, 39.95, 40.02 mm. Calculate the achieved accuracy and the repeatability within the 95% confidence interval. What is the resolution of the caliper shown above?

**Problem 1.27: Chain dimensioning**

The illustration below shows a simple drawing of a part made by milling. Calculate the symmetrical tolerance interval of A with 99% probability, assuming all tolerances are normally distributed within the  $\pm 3\sigma$  interval and independent of each other.

**Problem 1.28: Precision and accuracy of a stepper motor**

Consider a linear motion system that consists of a NEMA 17 size hybrid stepping motor connected to a trapezoidal drive spindle.



The NEMA 17-size hybrid stepping motor provides a  $1.8^\circ$  step angle (200 steps/revolution) suitable for a holding torque of 3.2 kg-cm (0.314 Nm). Standard stepper motors have an accuracy of  $\pm 5\%$  of a step. This means, that over a travel of many steps, the error in the actual travel is always less than 5% of a step.

The rotation of the stepper motor is transferred in linear motion using a ball screw with a lead of  $P_h = 3\text{mm}$ . The accuracy of the lead is graded according ISO with a category classification C1 with  $v_{300} = 5\text{ }\mu\text{m}$ . This means, that within a travel range of 300 mm, the error in the actual travel is less than 5  $\mu\text{m}$ .

- Calculate the resolution for linear motion.
- Calculate the repeatability interval for linear motion.
- Calculate the maximum offset of the linear motion system expressed as a percentage of the travel length, affected by the accuracy of the stepper motor. The maximum offset is the maximum value of the difference between the intended position and the achieved position.

### Problem 1.29: Load induced error from shaft coupling

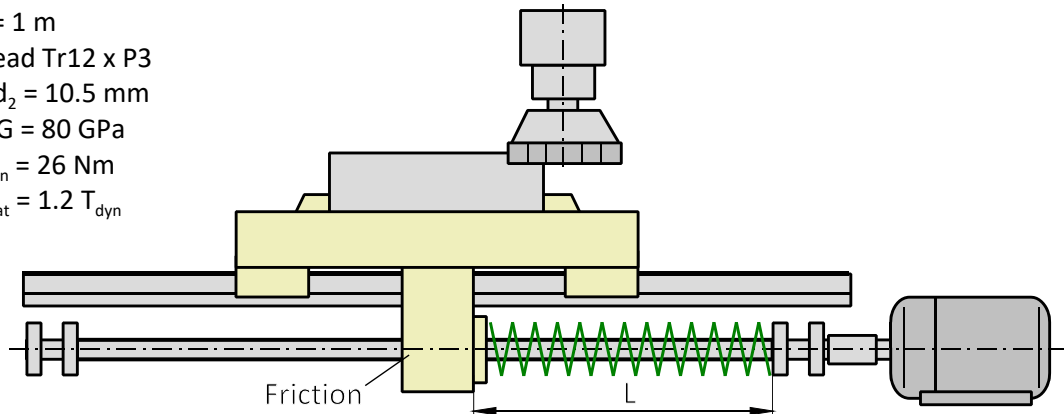
Consider the linear motion system as described in the problem above. The stepper motor is connected to the lead screw using a flexible coupling. The torsional stiffness of the coupling is  $k_\phi$  [Nm/rad] the axial stiffness of the coupling  $k_a$  [N/m]. One of the bearings in the stepper motor is locating. The bearing on the left side of the spindle is non-locating. Further consider a drive torque of  $M_\phi = 0.1 F_l d/2$ .

Calculate the ratio  $s_\phi / s_a$ , where  $s_\phi$  is the partition of the load induced error as result of the torsional flexibility of the coupling and  $s_a$  is the partition of the load induced error as a result of the axial flexibility of the coupling.

### Problem 1.30: Hysteresis error from friction in the drive spindle

In the figure below a linear motion axis of a milling machine is shown actuated by a servo motor. The system accuracy suffers from the “Wind-up” of the lead screw. The “Wind-up” of drive shafts is defined as the torsion angle. It is assumed that the displacement of the carriage is set by the rotation angle of a stepper motor. The drive torque is present in one direction of motion.

Screw length  $L = 1$  m  
 Trapezoidal thread Tr12 x P3  
 Pitch diameter  $d_2 = 10.5$  mm  
 Shear modulus  $G = 80$  GPa  
 Drive torque  $T_{\text{dyn}} = 26$  Nm  
 Drive torque  $T_{\text{stat}} = 1.2 T_{\text{dyn}}$   
 $v = 0.1$  m/s



- Calculate the hysteresis error resulting from the friction in the drive spindle.
- Calculate the fluctuation and frequency of the stick-slip movement assuming that the driving torque is 20% higher by static friction, before the motion starts. The carriage is driven with  $v=0.1$  m/s (Hint: Figure 1.26 page 30).
- Calculate the displacement of the carriage by frictional heating in the nut-spindle interface. One of the bearings in the motor drive is the locating bearing of the drive spindle. Consider a mean value of temperature increase over the spindle of  $10^\circ\text{C}$ . The thermal expansion coefficient is  $\alpha=12 \cdot 10^{-6}$  /K.



**Problem 1.31: Hysteresis error from friction in the dovetail slide**

In the figure below a linear motion axis of a milling machine is shown actuated by a servo motor. The system accuracy suffers from the “friction in the dovetail slide” of the carriage and the resulting hysteresis error. It is assumed that the displacement of the carriage is set by the rotation angle of a stepper motor. The friction is present in both directions of motion.

Screw length  $L = 1$  m

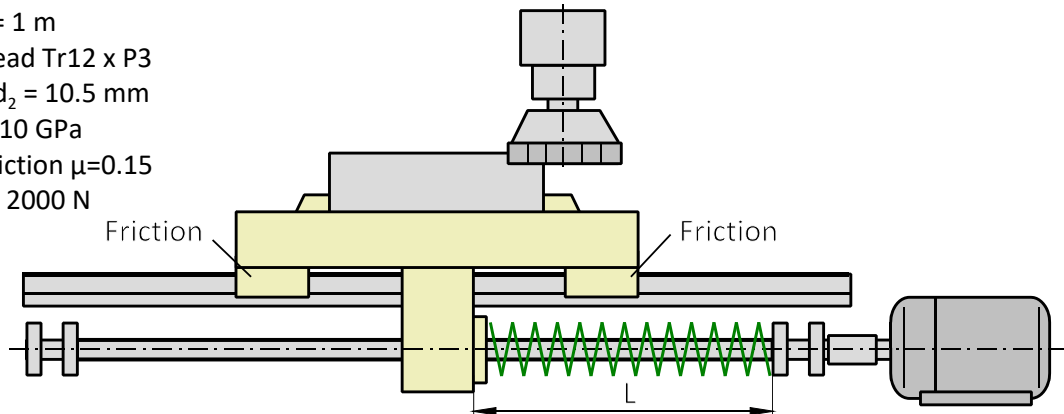
Trapezoidal thread Tr12 x P3

Pitch diameter  $d_2 = 10.5$  mm

E modulus  $E = 210$  GPa

Coefficient of friction  $\mu = 0.15$

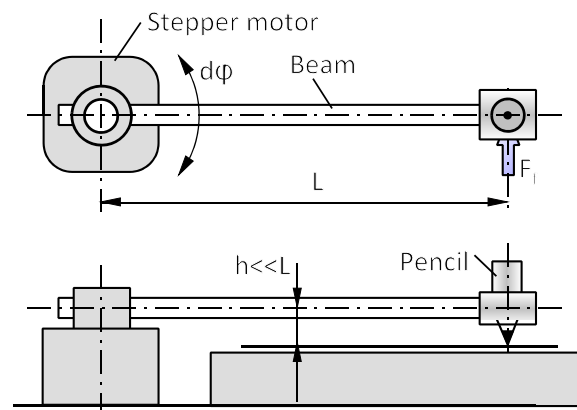
Normal load  $F = 2000$  N



- Calculate the hysteresis error resulting from the friction in the dovetail slide.
- Calculate the fluctuation- and frequency of the stick-slip movement that occurs if the coefficient of friction varies between  $\mu_s = 0.25$  and  $\mu_d = 0.1$  and the sliding velocity  $v = 0.1$  m/s (Hint: Figure 1.26 page 30).

**Problem 1.32: Hysteresis error in a robot arm**

Consider a simplified model of a robot arm that is actuated by a stepper motor using 200 steps/rotation. The beam length  $L$  measured from the shaft centre of the stepper motor to the point of contact between the pencil and the paper is  $L = 0.2$  m. The beam is made of steel with  $E = 210$  GPa. The beam has a diameter of  $d = 5$  mm and is considered to be loaded in pure bending. The friction force between the pencil and the paper is  $F_f = 2$  N.

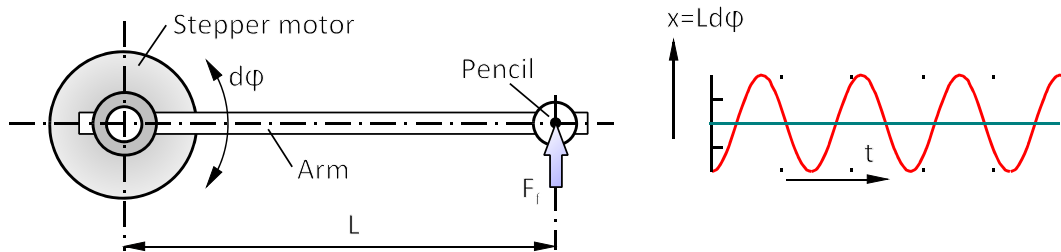


- Calculate the virtual play as a result of the elasticity of the robot arm. Hint: the virtual play is defined as the deflection resulting from a bidirectional motion.
- Calculate the resolution [mm] of the movement as a result of the stepper motor. Hint: The resolution is the smallest incremental step size.
- Calculate the fluctuation  $\Delta X$  of the stick-slip movement that occurs, when the friction force varies between  $F_s = 5$  N and  $F_d = 2$  and the sliding velocity  $v = 0.1$  m/s (Hint: Figure 1.26 page 130)



### Problem 1.33: Virtual play error in a robot arm

Consider the robot arm described in previous problem. The robot arm is actuated in such a way that the pencil should displace  $Ld\phi = \pm 5$  mm. It is considered that the top-top value of the movement is  $x = \pm 4$  mm.



a) If the error between actuation and obtained displacement is explained by virtual play. How much would the virtual play be?

### Problem 1.34: Virtual play in drive spindle assembly

In the figure below a lead screw with anti-backlash split nut is shown. The assembly consists of a 303 stainless steel acme lead screw mated with a self-lubricating polyacetal (POM with 15% PTFE) nut. The spring preload eliminates backlash and compensates for wear.

Spring preloaded systems can generally transfer the actuation force only in one direction. For bidirectional actuation the spring preload must exceed the actuating force. The large preload that is required causes extra friction, wear and increases the hysteresis error of the motion system.

Consider a motion system where the displacement is set by the rotation angle of a stepper motor. The system accuracy suffers from the wind-up of the lead screw. The wind-up is defined as the torsion angle. It is asked to calculate the hysteresis error for a bidirectional motion as a result of the spring preload.

Screw length  $L = 1$  m  
 Trapezoidal thread  $Tr12 \times P3$   
 Pitch diameter  $d_2 = 10.5$  mm  
 Shear modulus  $G = 80$  GPa  
 Coefficient of friction  $\mu = 0.15$   
 Internal spring preload  $F_i = 500$  N



a) Calculate the friction torque of the combined split nut.

b) Calculate the hysteresis error for a bidirectional motion with zero payload and a frictional torque of  $T = 1$  Nm.

**Problem 1.35: Amplification factor of load induced error**

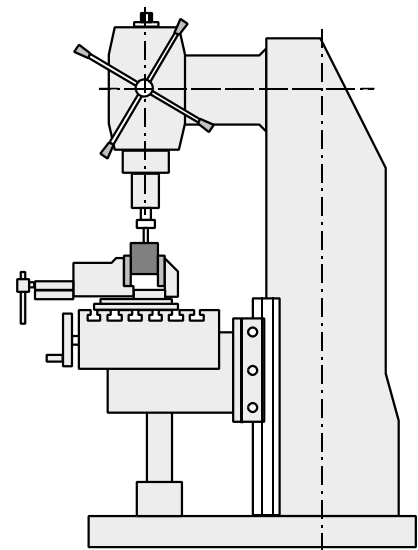
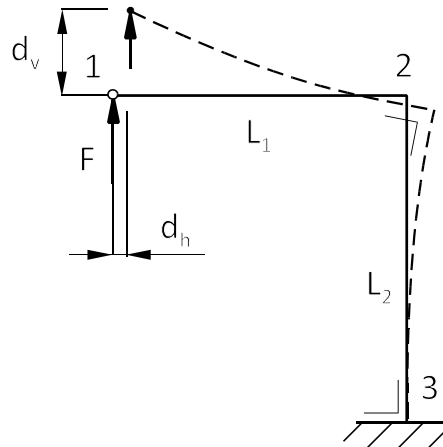
Many cutting machines have massive structures to compensate for the low stiffness that “open structures” suffer from.

Consider a structure with two beams of unequal length  $L_1$  and  $L_2$  in the position as illustrated.

The displacement  $d_v$  by elastic deflection of the whole structure can be expressed as

$$d_v = X \frac{FL_1^3}{3EI_{x,1}}$$

where  $X$  is a so called amplification factor. Derive an equation for the amplification factor  $X$  as a function of the length ratio and the geometrical moments of inertia of the beams.

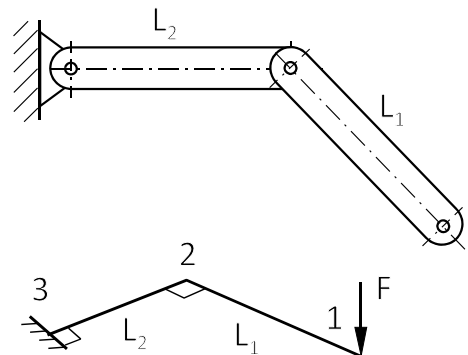
**Problem 1.36: Amplification factor of load induced error**

Consider a planar mechanism shown in the figure. Consider both beams at right angle, loaded with  $F$ . In this position beam  $L_1$  is loaded in pure bending, beam  $L_2$  is loaded in bending and torsion.

The displacement  $d_v$  by elastic deflection in point 1 can be

expressed as 
$$d_v = X \frac{FL_1^3}{3EI_{x,1}}$$

where  $X$  is the amplification factor. Derive an equation for the amplification factor  $X$ .



### Problem 1.37: Stick slip motion

A mass of  $m=0.5$  kg is pulled along a surface using a spring balance. The spring balance is moved with a constant velocity of  $v=0.1$  m/s. The motion exhibits a clear stick-slip behaviour. The reading of the spring balance varies between 8 N and 14 N. The stiffness of the spring  $k = 1$  N/mm.

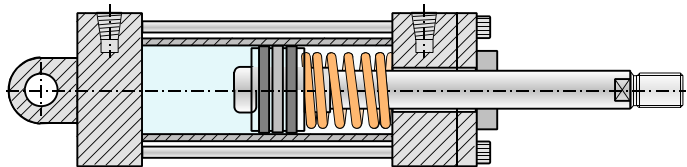


- Calculate the ratio between the static and dynamic coefficient of friction.
- Calculate the fluctuation of stick-slip motion.
- Calculate the frequency of motion.
- Calculate the natural frequency of the mass spring system.

### Problem 1.38: Stick slip motion

One of the options in a morphological matrix to fulfil the actuator function in a linear motion drive system is a pneumatic cylinder. The friction between the piston seals and the cylinder typically results in a friction as modelled in Figure 1.26 page 30.

The seal friction is highest in the static position of the piston and falls down to a very low level by elasto-hydrodynamic lubrication when the piston moves.



Consider a starting position with an air pressure equal to a spring force  $F_0 = 500$  N (zero friction). The spring stiffness  $k=10$  N/mm. Next the air pressure is increased until the piston starts to move. The piston starts to move when the air pressure has increased by 10%. When the piston starts to move the friction falls down to approximate zero level. The piston will make a step forward. This process is repeated.

Calculate the smallest possible step size in a step by step movement of the piston.

### Problem 1.39: True or Untrue?

Read the related chapter of the course book before answering the true / untrue statements. These statements are useful to verify whether you understand the theory discussed in this chapter.

Key words: Quality, reliability, failure mode, - mechanism, - cause, precision, accuracy, resolution, backlash, virtual play...

- Quality is how well a product conforms to requirements at one point in time, the moment of quality control (page 40).
- If high precision is a requirement of a measuring device, than a good quality measuring device has high precision (page 40).
- Reliability  $R(t)$  is equivalent to predictability (page 40). When the quality of a positioning device degrades over time by wear, the precision degrades and measured data becomes less reliable.
- If the clamping force of an interference fit using polymer components might fail by creep (requirement is to transmit a torque), then creep is the failure mode (page 40).

5. If the clamping force of an interference fit using polymer components might fail by creep, then the loss of clamping force is the failure mechanism (page 40).
6. The key functions of a rolling bearing are to enable free movement in the intended degree of freedom and infinite stiffness in the others. Possible failure modes of rolling bearings are jamming or shaft runout by vibrations (page 40).
7. Surface fatigue is a possible failure mode of a rolling bearing (page 40).
8. The failure mode of a bearing caused by surface fatigue might be the noise / level of vibrations that increases by surface fatigue.
9. The failure mechanism that caused a failure mode can be rolling contact fatigue, wear etc. (page 40).
10. Possible failure mechanisms of rolling bearings are surface fatigue and elastic indentation of the balls in the raceways. Both failure mechanisms increase the noise level by vibrations (page 40).
11. Runout and vibrations in rotating bearings create an offset of the shaft center. The RMSE of the offset is defined as the running accuracy (page 22).
12. The root causes of failures can in theory in most cases be traced back to some kind of a human error, e.g. design failure, operational failure, maintenance induced failure, specification failure etc. (page 40).
13. If an unexpected high load causes a component to fail than the unexpected high load is the failure mode (page 40).
14. If the accuracy of a linear axis degrades by wear, then loss of accuracy is the failure mode and wear is the failure mechanism. The cause of failure can be an improper material selection.
15. Positioning error of a linear axis is the difference between the intended position of a specific point of interest and the measured position. The accuracy can be expressed in the RMSE of a number of attempts (page 22).
16. Precision (Repeatability) is the mean value of the positioning errors of a number successive attempts to move the machine to the same position (page 22).
17. Resolution is the smallest possible movement of a system when actuated. Also known as the step size (page 22).
18. Bi-directional repeatability is the precision of an XY stage (page 22).
19. Backlash is defined as the virtual play that a mechanical structure exhibit when actuated by reversed loading (page 22).
20. Machining tolerances on the pitch P of a screw spindle of a motion drive results in less precise positioning of the slider.
21. The load induced error results in a less precise positioning.
22. The end support bearing of the screw spindle of a motion drive using a rigid coupling between the stepper motor and the drive spindle does not need to be locating.

## Answers

1.1)

$$\mu = \frac{1 \cdot 6 + 2 \cdot 7 + 4 \cdot 8 + 2 \cdot 9 + 1 \cdot 10}{n} = 8 \text{ months}$$

$$\sigma = \sqrt{\frac{(\mu-6)^2 + 2(\mu-7)^2 + 4(\mu-8)^2 + 2(\mu-9)^2 + (\mu-10)^2}{n-1}} = 1.155 \text{ months}$$

$$L_{10}: \quad z=1.28 \quad x = \mu - z \sigma \quad x = 6.522 \text{ months}$$

1.2)

The mean value of the diameter of the shafts  $\mu=19.974$  mm. The 99.7% interval is a  $\pm 3\sigma$  interval which results in  $\sigma = 52 \mu\text{m}/6$ . The 95% symmetrical interval, with 2.5% on either side, is between  $\mu-z\sigma$  and  $\mu+z\sigma$  with  $z=1.96$ . This results in a tolerance interval of  $2z\sigma = 34 \mu\text{m}$ .

1.3)

20H7:  $D_{\max} = 20 \text{ mm} + 21 \mu\text{m}$ ,  $D_{\min} = 20 \text{ mm} + 0 \mu\text{m}$ ,  $D_{\text{mean}} = 20 \text{ mm} + 10.5 \mu\text{m}$ ,  $\sigma_D = 21/6 \mu\text{m}$   
 20r6:  $d_{\max} = 20 \text{ mm} + 41 \mu\text{m}$ ,  $d_{\min} = 20 \text{ mm} + 28 \mu\text{m}$ ,  $d_{\text{mean}} = 20 \text{ mm} + 34.5 \mu\text{m}$ ,  $\sigma_d = (41-28)/6 \mu\text{m}$

Interference mean  $\delta_{50} = 34.5 \mu\text{m} - 10.5 \mu\text{m} = 24 \mu\text{m}$ , Interference SD  $\sigma = \sqrt{(\sigma_D^2 + \sigma_d^2)} = 4.11 \mu\text{m}$

The minimum value of  $\delta$  with 1% failure probability  $\delta_1 = \delta_{50} - z \sigma = 24 - 2.33 \cdot 4.11 = 14.41 \mu\text{m}$

Coefficient of variation, probabilistic:  $CV' = \frac{\delta_{50} - \delta_1}{\delta_{50}} = 0.4$  thus,  $T_{\min} = 0.6 T_{\text{mean}}$

The worst case scenario is obtained with the maximum bore diameter and the minimum shaft diameter.

The minimum value of  $\delta$  (Deterministic):  $\delta_{\min} = D_{\max} - d_{\min} = 7 \mu\text{m}$  and  $CV' = \frac{\delta_{50} - \delta_{\min}}{\delta_{50}} = 0.71$

1.3b)

What if two variables are multiplied, for example the torque proportional to the product of the interference  $\delta(\mu, \sigma) = (40, 4)$  and the friction  $\mu_f(0.2, 0.02)$ .

(Reference: <http://www.wisfaq.nl/show3archive.asp?id=15989&j=2003>)

$$\mu_z = \mu_x \mu_y \quad \sigma_z^2 = \sigma_x^2 \sigma_y^2 + \sigma_x^2 \mu_y^2 + \sigma_y^2 \mu_x^2$$

Note that the CV-value of both parameters individually is 10%. The CV-value of the combined variables has increased to 13.4%

1.4)

The mean value of the coefficient of friction  $\mu_{50} = 0.14$ . The obtained preload  $F$ , with given tightening torque  $M$ , is minimal when the friction in fastener will have the highest value.

$$\text{Torque ratio (worst case): } T = c \frac{M_a}{\mu} \quad T \propto \frac{1}{\mu} \quad CV' = \frac{1/\mu_{\text{mean}} - 1/\mu_{\text{max}}}{1/\mu_{\text{mean}}} = 0.125$$

where  $T$  is the torque that can be transmitted by the clamping action between the shaft - hub interface. Compared to the interference fit which is discussed in previous problem, a much smaller variation in the torque is obtained, and thus a more reliable connection is achieved. By controlling the friction, the reliability of the tapered shaft hub connection can be improved further. Besides special lubricants in the

threaded area, washers are occasionally used as a means of minimising frictional scatter from the heat face friction during tightening.

#### 1.4b)

What if two variables are multiplied, for example the torque proportional to the product of the interference  $\delta(\mu, \sigma) = (40, 4)$  and the friction  $(0.2, 0.02)$ .

(Reference: <http://www.wisfaq.nl/show3archive.asp?id=15989&j=2003>)

$$\mu_{xy} = \mu_x \mu_y \quad \mu_{xy} = 40 \cdot 0.2 = 8$$

$$\sigma_{xy}^2 = (\sigma_x^2 \mu_y^2) + (\sigma_y^2 \mu_x^2) - \mu_x^2 \mu_y^2 \quad \sigma_{xy}^2 = \sigma_x^2 \sigma_y^2 + \sigma_x^2 \mu_y^2 + \sigma_y^2 \mu_x^2$$

$$\sigma_{xy}^2 \approx \sigma_x^2 \mu_y^2 + \sigma_y^2 \mu_x^2 \quad \sigma_{xy}^2 \approx 4^2 \cdot 0.2^2 + 0.02^2 \cdot 40^2 = 1.28 \quad \sigma_{xy} \approx 1.13$$

$$CV = \frac{\sigma_{xy}}{\mu_{xy}} = \frac{1.28}{8} = 14\%$$

Note that the variability of both parameters individually was 10%. The variability of the combination has increased somewhat.

#### 1.5)

A small variation in the torque transmission is found, despite the large tolerance field on the diameter of the components.

$$\text{Torque ratio (worst case): } CV' = \frac{T_{50} - T_{\min}}{T_{50}} = \frac{p_{50} - p_{\min}}{p_{50}} = \frac{\sigma(\varepsilon = 0.0075) - \sigma(\varepsilon = 0.005)}{\sigma(\varepsilon = 0.0075)} = \frac{400 - 350}{400} = 0.125$$

#### 1.6)

$$\text{a) } \mu' = \frac{200 \cdot 0.15 + 250 \cdot 0.25}{2} \quad \mu = 46.25 \quad CV = \frac{|\mu - 200 \cdot 0.15|}{\mu} = 35\%$$

b)

$$\mu_x = 0.2 \quad \sigma_x = \frac{0.05}{3} \quad \mu_y = 225 \quad \sigma_y = \frac{25}{3} \quad \mu_{xy} = \mu$$

$$\sigma_T = a \sigma_{xy} \quad \sigma_{xy} = \sqrt{\sigma_x^2 \sigma_y^2 + \sigma_x^2 \mu_y^2 + \sigma_y^2 \mu_x^2} \quad CV = \frac{3\sigma_T}{\mu_T} = \frac{3\sigma_{xy}}{\mu_{xy}} = 26.6\%$$

#### 1.7)

Consider single side truncation of the probability density function

$$x_1 = \mu - z\sigma, \quad z = 2.33 \rightarrow x_1 = 103 \cdot 10^3 \text{ km}$$

$$x_{10} = \mu - z\sigma, \quad z = 1.28 \rightarrow x_{10} = 124 \cdot 10^3 \text{ km}$$

Note that with a 20% longer service the failure probability will increase to 10%.

#### 1.8)

The Mean Time Between Failure MTBF =  $T/r = T_{\text{test}} \cdot n / r = 10 \cdot 500 / 2 = 2500 \text{ hr / failure}$

The failure rate  $\lambda$  is the inverse of the MTBF,  $\lambda = 1/\text{MTBF} = 1 / 2500 = 0.04\% / \text{hr}$

The reliability  $R(t) = \exp(-\lambda t)$  according (eq. 1.4 page 12),  $R(\text{MTBF}) = \exp(-1) = 0.37 = 37\%$

The probability that the component will work for 50% of the MTBF is  $R(t)=\exp(-0.5)=0.61$   
 $R(t)=0.95$  is obtained with  $-\ln(0.95)=0.0513$  MTBF, only 5% of the MTBF.

$$1.9) \quad F(t) = 1 - A \cdot B \cdot C \cdot [1 - (1-D)(1-E)] [1 - (1-F)(1-G)] = 0.32$$

$$1.10) \quad L_{10h} = \frac{L_{10}}{60n} = 2083\text{hr} \quad L_{nh} = 1000\text{hr} \quad a_1 = \frac{L_{nh}}{L_{10h}} = 0.48 \quad R = \exp(-(a_1/4.48)^{3/2}) = 0.966$$

$$1.11) \quad R_s = 0.99 \quad n = 3 \quad R_j = 1 - (1 - R_s)^{1/n} \quad R_j = 0.785 \\ L_{nh} = a_1 L_{10h} \quad a_1 = 4.48 \ln(1/R_j)^{2/3} \quad a_1 = 1.743 \\ L_{10h} = 5738\text{hr} \quad n = 4000\text{rpm} \quad L_{10} = 1377 \cdot 10^6 \\ C/P = (L_{10}/10^6)^{1/3} = 11.1 \quad P/C = 0.09$$

1.12)

Standard deviation  $\sigma = 0.08\mu$

$L_n = \mu - z\sigma$ , 99% reliability,  $n=1$ ,  $z=2.33$

$L_{50} = \mu$ , 50% reliability

$C_{\text{reliab}} = L_n/L_{50}$ ,  $L_1/L_{50} = (\mu - z \cdot \sigma)/\mu = 0.814$

Reliability factors  $C_{\text{reliab}}$  for  $\sigma=0.08\mu$

R(t)	50%	90%	99%	99.9%
$C_{\text{reliab}}$	1.000	0.897	0.814	0.753

$$1.13) \quad x = L_n, \mu = L_{50}, \sigma = 0.2\mu \quad z = \frac{\mu - x}{\sigma} \quad z = 1.667 \quad \text{Table 1.2: } R = 0.95$$

1.14)

A chain with 100 components (links) connected in series and component reliability 0.99 results in a system reliability of  $0.99^{100}=0.366$ . If, one of the components has component reliability 0.95 (the weakest link in the chain), the system reliability becomes  $0.99^{99} \cdot 0.95=0.351$ . The conclusion is that "A chain is only as strong as its weakest link" is valid in a deterministic approach, and it is the criterium for failure by overload. In the case of a probabilistic approach which is necessary for reliability analysis if the failure mode is fatigue, the reliability of all components matter.

$$1.15) \quad x=100, \mu_1=150, \sigma_1=30.5, \mu_2=120, \sigma_2=10.2$$

$$z_i = \frac{\mu_i - x}{\sigma_i} \quad z_1 = 1.64 \quad z_2 = 1.96 \quad \text{Table 1.2: } R_1 = 0.95 \quad R_2 = 0.975 \quad R_s = R_1 R_2 \quad F_s = 1 - R_s$$

$$1.16) \quad x=150, \mu=200, \sigma=0.2\mu \quad z = \frac{\mu - x}{\sigma} \quad z = 1.25 \quad \text{Table 1.2: } R = 0.9 \quad R_s = R^4 = 66\%$$

1.17)

$$C = 7.28\text{ kN}, P = 0.1C, L_{10} = (C/P)^3 10^6 = 10^9 \text{ rev}$$

$$L_{10h} = L_{10}/60n = 11.9 \cdot 10^3 \text{ hr} = 6.2 \text{ year}$$

$$L_1 = a_1 L_{10h}, a_1 = 0.21, L_{1h} = 1.3 \text{ year}$$

The ball bearing loaded with  $P=0.1C$  will have a life expectancy with 1% failure probability of 1.3 year.

1.18)



a)

$$\left. \begin{aligned} a_1 &= \frac{L_{nh}}{L_{10h}} = \frac{10,000}{16,000} = 0.625 \rightarrow R_1(10,000hr) = 0.949 \\ a_1 &= \frac{L_{nh}}{L_{10h}} = \frac{10,000}{12,000} = 0.833 \rightarrow R_2(10,000hr) = 0.923 \end{aligned} \right\} R_s = R_1 R_2 = 0.876$$

b) Weibull's reliability function is (eq. 1.5 page 12)

$$R(t) = \exp\left(-\left(\frac{t}{\theta}\right)^\beta\right) \text{ solving } \theta \text{ results in: } \theta = \frac{t}{(-\ln(R))^{1/\beta}}$$

Consider bearing 1: Substitution of  $\beta=1.5$ ,  $R=0.9$  and  $t=L_{10h}=16000$  hr results in  $\theta=71724$  hr  
The reliability of bearing 1 for the life of  $t=10000$  hr can now be solved

$$R(t=10000) = \exp\left(-\left(\frac{t}{\theta}\right)^\beta\right) = 0.949$$

Repeating the method above for the other bearing result in the system reliability:

$$R_2 = 0.923, R_{system} = R_1 R_2 = 0.876$$

**1.19)**

A = 0.95, B = 0.9, C = 0.8, D = 0.7

$$R_{CD} = 1 - (1 - C)(1 - D) = 0.94$$

$$R_{unit} = A B (1 - (1 - C)(1 - D)) = 0.804$$

$$R_{system} = R_{unit}^6 = 0.27$$

The reliability of the components A and B dominate the system reliability.

**1.20)**

$$\mu = \frac{590+420+520+480+\dots}{8} = 492.5$$

$$\sigma = \sqrt{\frac{(\mu-590)^2 + (\mu-420)^2 + (\mu-520)^2 + \dots}{n-1}} = 50.64$$

$$99\% \text{ reliability; } z=2.33 \quad t = \mu - z \sigma = 375 \text{ hr}$$

**1.21)** True or untrue statements are discussed in the lectures and referred to the book.

**PRECISION:**

**1.22)** Repeatability (precision) is the error between a number of successive attempts to move the machine to the same position. The larger the standard deviation the smaller the repeatability (precision).

**1.23)**

$$a) \quad \mu = \frac{1}{n} \sum_{i=1}^n x_i = 99.6 \quad \sigma = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \mu)^2} = 0.267 \quad z=1.96 \quad z\sigma = 0.524$$

Note: 95% interval, 2.5% on each side,  $z=1.96$

$$b) \quad \text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - x_{\text{target}})^2} = 0.472$$

$$c) \quad \text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i + \delta - x_{\text{target}})^2} = 0.25$$

Note: The accuracy cannot further be improved without increasing the precision of the plotter.

**1.24)** Bending of the pencil:  $\delta_p = \frac{F_f}{k} = 2 \text{ mm}$

Bending of the beam:  $\delta_{\text{bending}} = \frac{F_f L^3}{3EI_x} \quad I_x = \frac{\pi}{64} d^4$

Torsion of the beam:  $\delta_{\text{torsion}} = \phi h \quad \phi = \frac{TL}{GI_p} \quad I_p = \frac{\pi}{32} d^4 \quad T = F_f h$

flexibility of the beam / flexibility of the pencil =  $\frac{\delta_{\text{torsion}} + \delta_{\text{bending}}}{\delta_p} = \frac{1.83 + 0.83}{2} = 1.33$

**1.25)**

The mean value is in this case the same as the median,  $\mu=0.13$ .

$$\mu = \frac{0.11+0.12+0.13+0.14+0.15}{n} = 0.13$$

$$\sigma = \sqrt{\frac{(\mu-0.11)^2 + (\mu-0.12)^2 + (\mu-0.13)^2 + (\mu-0.14)^2 + (\mu-0.15)^2}{n-1}} = 0.016$$

The 95% interval of the COF of friction becomes  $COF = \mu \pm 1.96\sigma = 0.13 \pm 0.031$

The standard deviation of the sample mean is  $\sigma_m = \frac{\sigma}{\sqrt{n}} = 0.0071$

The 95% reliability interval of the sample mean is  $COF = \mu \pm 1.96\sigma_m = \pm 0.014$

The number of experiments needed to have a 95% reliability of the estimation of the mean within  $\pm 0.01$  accuracy is:

$$0.01 = 1.96 \frac{\sigma}{\sqrt{n}} \quad n = \left( \frac{1.96\sigma}{0.01} \right)^2 \quad n = 10$$

### 1.26)

$N=5$ ,  $\mu = 40.028$  mm,  $\sigma = 0.068$  mm, RMSE = 0.067 mm

a) Accuracy: 0.067

b) Precision can be represented by the interval that contains N% of the measured positions. Precision within a 95% confidence interval:  $\pm 1.96\sigma = \pm 0.134$  mm

c) Resolution of the caliper is 0.01 mm

95% confidence interval of accuracy:  $\pm 1.96\sigma_m$ ,  $\sigma_m = \sigma/\sqrt{n} = 0.031$ ,  $COF = \pm 0.062$

number of experiments needed to determine the accuracy within a 95% confidence interval of  $\pm 0.05$

$$0.05 = 1.96\sigma/\sqrt{n}, \quad n = (1.96\sigma/0.05)^2 = 7$$

### 1.27)

$$\mu_{15} = 15 \text{ mm} \quad \mu_{40} = 40 \text{ mm} \quad \mu_{70} = 70 \text{ mm}$$

$$\sigma_{15} = \frac{0.2 \text{ mm}}{3} \quad \sigma_{40} = \frac{0.1 \text{ mm}}{3} \quad \sigma_{70} = \frac{0.2 \text{ mm}}{3}$$

$$\mu_x = \mu_{70} - \mu_{40} - \mu_{15} \quad \mu_x = 15 \text{ mm}$$

$$\sigma_x = \sqrt{\sigma_{15}^2 + \sigma_{40}^2 + \sigma_{70}^2} \quad \sigma_x = 0.1 \text{ mm}$$

99% interval, 0.5% on both sides:  $z = 2.58$

$$x_{\min} = \mu_x - z\sigma_x \quad x_{\max} = \mu_x + z\sigma_x \quad A = z\sigma_x = 0.258 \text{ mm}$$

A rule of thumb says that with chain dimensioning the probabilistic value can be estimated by the square root of the tolerances. This would result in:  $A = \sqrt{(0.2^2 + 0.1^2 + 0.2^2)} = 0.3$

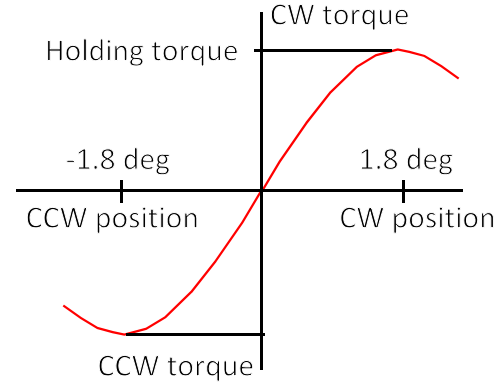
### 1.28)

a) Resolution stepper motor  $360/200 = 1.8^\circ$ , resolution for linear motion  $Ph/200 = 0.015$  mm

b) The geometrical error will affect the offset, but not the repeatability. The offset will be repeatable when the slide is directed for several times to the same position.

c) The maximum offset as a result of the stepper accuracy is  $\pm 5\% \cdot 360/200 = \pm 0.09^\circ = 0.18^\circ$ . For linear motion  $\pm 0.05 \cdot Ph/200 = \pm 0.75 \mu\text{m}$ . The offset as a result of the screw accuracy is  $\pm 5 \mu\text{m}$ . The accuracy of the motion system is  $5.75 \mu\text{m}$ .

Note: The stepper motor makes full steps when unloaded. When loaded, the drive torque will result in some shaft rotation. The load versus rotation is approximated by a mechanical torsion spring with a sinusoidal relationship. The holding torque (stall torque) will result in  $1.8^\circ$  rotation. From this it is approximated that a static torque load equal to 15% of the holding torque will displace the motor shaft one tenth of a full step from the origin.



1.29) error in motion as a result of the finite axial stiffness:  $s_a = \frac{F_i}{k_a}$

error in motion as a result of the finite torsional stiffness:  $s_\phi = P_h \frac{\phi_i}{2\pi}$   $\phi_i = \frac{M_i}{k_\phi}$   $M_i = 0.1 F_i \frac{d_{as}}{2}$

error as a result of torsional stiffness and axial stiffness:  $\frac{s_\phi}{s_a} = P_h \frac{0.1 F_i \frac{d_{as}}{2}}{2\pi k_\phi} / \frac{F_i}{k_a} = \frac{0.1}{2\pi} P_h \frac{d_{as}}{2} \frac{k_a}{k_\phi}$

Note: The total axial deflection is  $s_a + s_\phi = s_a (1 + \frac{s_\phi}{s_a}) = X s_a$  where X is de amplification factor.

1.30)

$$a) \quad I_p = \frac{\pi}{32} d_2^4 \quad \phi = \frac{T_{dyn} L}{G I_p} = 0.272 \text{ rad} \quad s_v = \frac{\phi}{2\pi} P = 0.13 \text{ mm}$$

$$b) \quad \Delta M = 2 \cdot 20\% \cdot 26 \text{ Nm} = 10.4 \text{ Nm} \quad \Delta X = 2 \cdot 20\% \cdot 0.13 = 0.052 \text{ mm} \quad T = \Delta X / v \quad f = 1/T = 1.9 \text{ kHz}$$

Amplitude of motion is  $\Delta X/2 = 0.026 \text{ mm}$

The same result can be found by considering the velocity in the screw nut interface:

$$v_{carriage} = 0.1 \text{ m/s}, P = 3 \text{ mm}, n = v_{carriage} / P, n = 2000 \text{ rpm}$$

$$v_{contact} = \pi d_2 n = 1.1 \text{ m/s}, T = \frac{(d_2/2) \Delta \phi}{v_{contact}}, f = \frac{1}{T} = 1.9 \text{ kHz}$$

$$c) \quad \Delta L = L \alpha \Delta T = 0.12 \text{ mm}$$

1.31)

$$a) \quad \sigma = \frac{F_f}{A} \quad A = \frac{\pi}{4} d^2 \quad d = 10.5 \text{ mm} \quad \sigma = 3.465 \text{ MPa} \quad dL = L \frac{\sigma}{E} = 0.016 \text{ mm} \quad s_v = 2dL = 0.033 \text{ mm}$$

$$b) \quad k = \frac{AE}{L} = 18.2 \cdot 10^6 \text{ N/m} \quad \Delta F = 2P(\mu_s - \mu_d) = 600 \text{ N} \quad \Delta X = \frac{\Delta F}{k} = 0.033 \text{ mm} \quad \Delta t = \frac{\Delta X}{v} \quad f = \frac{1}{\Delta t} = 3 \text{ kHz}$$

1.32) Virtual play:  $\delta = \frac{FL^3}{3EI}$   $s_v = 2\delta$

b) Resolution:  $d\phi = 360^\circ/200 \text{ steps} = 1.8^\circ$   $Ld\phi = 6.28 \text{ mm}$

Note that the resolution is not influenced by the virtual play. Explained in other words: The incremental step size is not influenced by the load induced error if the load is held constant.

c) Stick-slip fluctuation of movement:

$$k = \frac{F}{\delta} = 2.4 \cdot 10^3 \text{ N/m} \quad \Delta F = 2(F_s - F_d) = 6 \text{ N} \quad \Delta X = \frac{\Delta F}{k} = 2.48 \text{ mm} \quad \Delta t = \frac{\Delta X}{v} \quad f = \frac{1}{\Delta t} = 40.3 \text{ Hz}$$

1.33) Virtual play is 2mm. The hysteresis error is the difference in motion that a mechanical structure exhibit by the reversal of travel direction.

1.34)

a) The tightening torque  $M_G$  of one half of the split nut is calculated using

$$M_G = F_i \frac{d_2}{2} \tan(\phi \pm \rho') \quad \tan(\phi) = \frac{P}{\pi d_2} \quad \rho' = \arctan\left(\frac{\mu}{\cos(\beta/2)}\right) \quad \beta = 30^\circ$$

The plus sign refers to tightening, the torque required to unscrew the other half of the split nut is calculated by using the minus sign. The total torque of the two halves of the split nut is

$$M_G = F_i \frac{d_2}{2} \tan(\phi + \rho') - F_i \frac{d_2}{2} \tan(\phi - \rho') \quad M_G = F_i \frac{d_2}{2} [\tan(\phi + \rho') - \tan(\phi - \rho')]$$

Note that the screw is switched from self locking to not self locking when  $\phi = \rho'$ . With a relative small coefficient of friction the preload on the second half of the split nut helps to reduce the torque. Substitution of the numerical data gives  $M_G = 0.822 \text{ Nm}$

b)  $I_p = \frac{\pi}{32} d^4$   $\phi = \frac{TL}{GI_p}$   $s_v = 2 \frac{\phi}{2\pi} P = 10 \cdot 10^{-3} \text{ mm}$

1.35)

$$d_v = \frac{FL_1^3}{3EI_1} + \theta_2 L_1 \quad \theta_2 = \frac{ML_2}{EI_2} \quad M = FL_1$$

$$d_v = \frac{FL_1^3}{3EI_1} + \frac{FL_1^2 L_2}{EI_2} = \left(1 + 3 \frac{L_2}{L_1} \frac{I_1}{I_2}\right) \frac{FL_1^3}{3EI_1}$$

1.36)

$$d_v = \frac{FL_1^3}{3EI_{x1}} + \theta_2 L_1 + \frac{FL_2^3}{3EI_{x2}} \quad \theta_2 = \frac{TL_2}{GI_{p2}} \quad T = FL_1$$

$$d_v = \frac{FL_1^3}{3EI_{x1}} + \frac{FL_1^2 L_2}{GI_{p2}} + \frac{FL_2^3}{3EI_{x2}} = \left(1 + \frac{L_2}{L_1} \frac{I_{x1}}{I_{p2}} \frac{3E}{G} + \left(\frac{L_2}{L_1}\right)^3 \frac{I_{x1}}{I_{x2}}\right) \frac{FL_1^3}{3EI_{x1}}$$

1.37)

a)  $F_{\max} = \mu_s F$   $F_{\text{mean}} = \frac{F_{\max} + F_{\min}}{2}$   $\mu_s = \frac{F_{\max}}{F}$   $\mu_d = \frac{F_{\text{mean}}}{F}$   $\frac{\mu_s}{\mu_d} = \frac{F_{\max}}{F_{\text{mean}}} = \frac{14}{11} = 1.27$

b)  $\Delta X = \frac{F_{\max} - F_{\min}}{k} = 6 \text{ mm}$   $\text{amplitude} = \frac{\Delta X}{2} = 3 \text{ mm}$

$$c,d) \quad T = \frac{\Delta X}{v} = 0.06 \text{ sec} \quad f = \frac{1}{T} = 16.7 \text{ Hz} \quad f_0 = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = 7.1 \text{ Hz}$$

**1.38)** Figure 1.26 page 30:  $X_0 = \frac{F_0}{k} = 50 \text{ mm}$   $X_1(1.1F_0) = 55 \text{ mm}$   $X_1 - X_0 = 5 \text{ mm}$

**1.39)** True or untrue statements are discussed in the lectures and referred to the book.

True 1, 2, 3, 6, 8, 9, 10, 11, 12, 14, 15, 17, 21, 22

Untrue 4, 5, 7, 13, 16, 18, 19, 20

**Chapter 2 - Problems with Solutions****Problem 2.1: Elongation at fracture**

A standardised tensile specimen made of a quenched and tempered steel grade C40 (material number 1.0511) is subjected to a tensile test.

- What percentage of strain (elongation) at fracture is expected?
- Would fracture be expected by material shear stresses or material tensile stresses.

**Problem 2.2: Failed tension bar**

A polished and heath treated (quenched) high carbon steel tensile specimen shows a yield strength of 1800 MPa and an ultimate tensile strength of 2200 MPa. A machined tension bar made of the same steel grade and subjected to the same heath treatment has fractured at a stress level of only 1500 MPa.

- Give a reasonable explanation for this much smaller strength?

**Problem 2.3: Drive shaft**

Consider a high carbon steel drive shaft loaded in pure torsion. The shaft is tempered at 200 degrees to relieve internal stresses generated during quenching without much loss of hardness. The shaft however remains quite brittle. Consider the next statement and motivate why it is true or untrue?

- When loaded up to fracture the fractured surface will be in the shear plane.

**Problem 2.4: Failed shaft hub connection**

A chain wheel is welded on a drive shaft. Both components are made of stainless steel. The reason for welding was that most alternative shaft hub connections have small clearances between the shaft and the hub which is more complicated with respect to cleaning (food industry).

- Consider the next statements and motivate why they are true or untrue?
  - This is typically a brittle fracture.
  - The shaft is fractured in the shear plane.
- What measures could be taken to improve the situation?



### Problem 2.5: Failed shaft subjected to rotary bending

Consider a quenched and tempered shaft that is subjected to rotary bending. The shallow groove visible in the picture below is cut into the shaft after heat treatment. The engineer that designed this shaft took into account the smaller surface area in the grooved section. The designer also took into account the stress concentration at the sharp corners in the grooved area.

Despite all effort in the design, the shaft has fractured in the grooved section at a much lower stress level than expected.



a) Explain the possible cause of this fracture that occurred at a stress level much lower than expected.

### Problem 2.6: Ceramic bar loaded in compression

Consider a ceramic bar loaded in compression until it fractures. Consider the next statements and decide whether it is true or untrue? The ceramic bar loaded in compression will fracture in the shear plane.

### Problem 2.7: Failed fastener of grade 10.9

a) A zinc plated fastener of steel grade 10.9 applied failed within one hour after applying a preload of  $F_i = 0.6F_{0.2}$  where  $F_{0.2}$  is the load at yield. Explain the possible cause of this failure. What's the name of this failure mechanism?

### Problem 2.8: Failed fastener of grade 316 stainless steel

a) A stainless steel fastener of grade 316 applied in the ceiling of a swimming pool failed after two years of service. Explain the possible cause of this failure. What's the name of this failure mechanism?

## Solutions

### 2.1)

- a) Elongation at fracture of 18% (slide 12 Level 1).
- b) Shear (slide 17 Level 1)

### 2.2)

- a) Quenched thus brittle, page 11 Level 2)

### 2.3)

- a) Brittle fracture is always in the tensile plane. It is at 45 degrees with the cross section for a shaft loaded in torsion.

**2.4)**

a) The fractured area shows “ratchet marks” over the circumference (Fig. 3.9). This is typically found for a fatigue fracture caused by rotary bending with stress concentrations in the circumference.

The stress concentrations are explained by the uneven root profiles, always present at the weld root. These stress concentrations become worse, by the irregular metal structure in the area of the weld toe.

The cross section is the tensile area when subjected to rotary bending.

The ratchet marks are visible over a large part of the cross section. If it was a brittle material only small cracks would already cause fracture.

Furthermore, the fractured surface looks polished, not grained. The polishing is generally caused by the rubbing surfaces during crack propagation.

b) It is a common design issue to locate the bearings at distance from the chain wheel. If this distance is minimized, the cyclic bending stresses become minimized or will vanish completely.

**2.5)**

a) Heat treated high carbon steel can be brittle, dependent on the post tempering. Brittle materials are sensitive to micro (roughness) and macro (geometry) stress concentrations (slide 20 Level 2).

What wasn't considered where the cracks that were initiated by cutting the groove in a quenched and tempered steel (from manufacturing using a tool with negative rake angle). Those cracks create stress concentrations larger than the stress concentration of the groove itself.

**2.6)**

The ceramic bar which is brittle will indeed fracture in the shear plane.

**2.7)**

a) Fasteners of steel grades higher than 8.8 are sensitive to hydrogen embrittlement (HE, HASCC). Hydrogen is often introduced during electroplating (galvanic plating) or electrochemical cleaning (etching).

**2.8)**

a) The protective corrosion layer on austenitic stainless steel is attached by chlorides and initiate SSC.

## Chapter 3 - Problems with Solutions

### Problem 3.1: Fatigue of a bicycle front fork

A manufacturer of mountain bike components developed a light weight magnesium alloy front fork. Unfortunately, the fork cracked during field testing under normal load conditions after only a few rough rides.

The design engineer was asked to check the fork design for stress concentrations. He made a solid model of the fork, performed 3D stress analysis in which the critical loading from braking and horizontal/vertical loading of the front wheel was set as input, optimized the model's surface mesh by creating a finer mesh around the small holes located at the brake connections and made the stress concentrations visible. The stress analysis proofed that the cracking experienced in the initial prototype testing was due to the part design, not the material properties of magnesium.

The first run showed areas of high stress beyond the critical failure point. The stress raisers were smoothened in some extra iterations, making modifications to the shape of the design and wall thickness. The modified design was manufactured and tested successfully.

The actual stresses in one of the critical area's are measured during one hour field testing. The stress spectrum is simplified into the values listed in the table below.

$\Delta\sigma_i$ [MPa]	240	120	60
$n_i$ [-]	10	20	40

Assume that the SN-curve of the magnesium alloy can be constructed with the reference points  $(\sigma_1, N_1)=(308\text{MPa}, 10^3)$ ,  $(\sigma_2, N_2)=(130\text{MPa}, 10^6)$  and  $(\sigma_3, N_3)=(88.5\text{MPa}, 10^8)$ .

- Calculate the m-values of the SN-curve on the left and right of the knee point.
- Consider the m-value on the left of the knee point  $m=8$  and on the right  $m=12$ . Calculate the fatigue life  $L$  that might be expected.
- Calculate the damage fraction  $D_1/D$  by the higher load of 240 MPa.

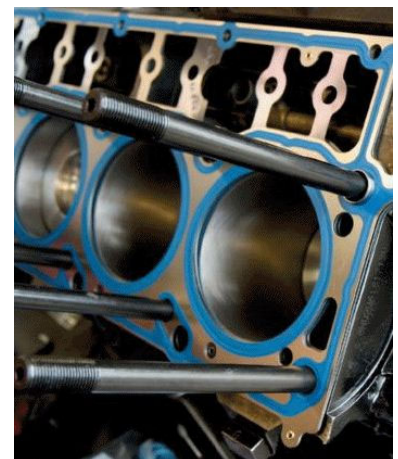


### Problem 3.2: Thermo Mechanical Fatigue (TMF) of cylinder head studs

The overall clamping force generated by the cylinder head bolts/studs and its uniform distribution across the entire sealing system is a major issue. Various areas are to be sealed (gas, water and oil seal) and compression forces and thermal expansion must be accommodated while keeping an optimum clamping force over the gasket.

Each time the motor temperature increases from cold start to operating temperature the aluminium cylinder head expands. Because the cylinder head studs are made of steel and thermally expand to a lesser degree, the bolt stress varies each temperature cycle.

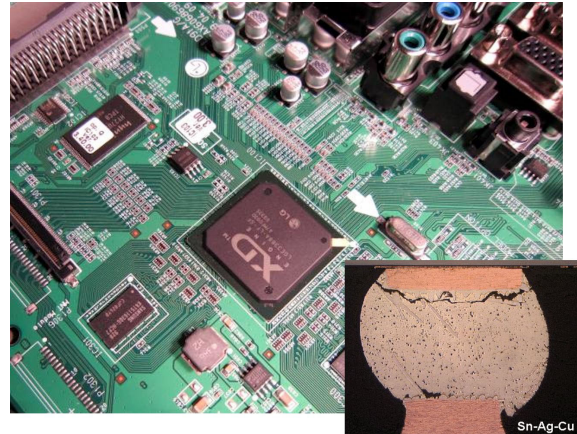
Calculate to what extent the bolt stress will increase by thermal expansion of the head. Consider  $d=10$  mm diameter bolt studs and a clamping length of  $L_m=150$  mm,  $E_b=210$  GPa, of  $\alpha_b=12 \cdot 10^{-6}/\text{K}$ ,  $dT=100$  K,  $E_m=70$  GPa,  $\alpha_m=23 \cdot 10^{-6}/\text{K}$  and  $k_m/k_b=11$ .



### Problem 3.3: Thermo Mechanical Fatigue (TMF) of solder joints

For environmental reasons tin-lead solders, for example Sn-37Pb, are replaced by lead free solders. Lead free solders, that are now being used are tin-silver-copper alloys. In the so called Ball Grid Array (BGA) packages in SMT these solder joints appear to suffer from Thermo Mechanical Fatigue.

The differences in the thermal expansion rates of the components and the printed circuit boards causes the solder joints to undergo cyclic elastic/plastic deformations. The plastic deformations can harden the solder and finally cause solder cracks and joint failure (cohesive bonding failure).



Derive an equation for the shear force acting on the solder joint, related to the thermo-mechanical properties of the components.

### Problem 3.4: Infinite life design cardan joint spline shaft loaded in torsion

Consider the cylindrical part of a drive shaft cyclically loaded in torsion. The ultimate tensile stress of the shaft is  $R_m = 500$  MPa. The shaft operates in the “infinite life” regime where  $\sigma'_e = 0.5 R_m$  (Table 3.1 page 80) and  $\sigma_e = 0.7 \sigma'_e$  (eq. 3.4 page 80) and  $\tau = 0.58 \sigma_e$  (von Misess). The yield stress of the shaft is  $R_{p0.2} = 0.6 R_m$  (non-alloy quality steel Quenched and Tempered, page 448).

Calculate  $D_{dyn}/D_{stat}$  where  $D_{dyn}$  is the shaft diameter required when dynamically loaded and  $D_{stat}$  when statically loaded.



- Stress concentrations are left out of consideration.
- The stress concentration in the shoulder fillet is  $K_t = 1.7$

### Problem 3.5: Infinite life design drive shaft with transverse hole loaded in torsion

Consider a drive shaft with transverse hole cyclically loaded in torsion with  $T = \pm 10$  Nm. The ultimate tensile stress of the shaft is  $R_m = 500$  MPa, the yield stress of the shaft is  $R_{p0.2} = 0.6 R_m$ . The diameter of the transverse hole in the shaft is related to the diameter of the shaft according  $d/D = 0.2$ . Calculate the diameter of the shaft when:

- statically loaded. Hint:

$$T = \frac{\pi}{16} D^3 \left(1 - 1.7 \frac{d}{D}\right)^3 \tau \quad \tau = 0.58 R_{p0.2}$$



- cyclically loaded in the “infinite life” regime where  $\sigma'_e = 0.5 R_m$  (Table 3.1 page 80),  $\sigma_e = 0.7 \sigma'_e$  and  $\tau_e = 0.58 \sigma_e$ . The stress concentration factor can be calculated with the curve fit function  $K_t = 1.5899 - 0.6355 \log(d/D)$ .

c) Consider a pin diameter  $d=0.2D$ . Calculate the ratio between the shear stress in the shaft from torsion and the shear stress in the pin.

### Problem 3.6: Fracture analysis of a pinned joint

Consider the fracture of the pin of a shaft-hub connection. The diameter of the cylindrical pin is related to the diameter of the shaft according  $d_{\text{pin}}/d_{\text{shaft}} = 0.2$ . The outer diameter of the hub is related to the diameter of the shaft according  $d_{\text{hub}}/d_{\text{shaft}} = 1.8$ . The pin is loaded in bending and shear.

a) Calculate the ratio between the bending stress  $\sigma_b$  and shear stress  $\tau$  in the cross section of the pin. Consider a uniform pressure distribution over the length of the hub-pin interface.



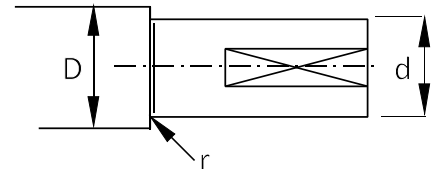
b) Calculate the maximum contact pressure between the shaft and hub.

### Problem 3.7: Infinite life design drive shaft under rotary bending

A stepped shaft is subjected to rotary bending  $M = 4 \text{ Nm}$ . The ultimate tensile stress of the shaft is  $R_m = 500 \text{ MPa}$ , the yield stress of the shaft is  $R_{p0.2} = 0.6R_m$ . Calculate the diameter of the shaft when:

a) statically loaded.

b) cyclically loaded in the “infinite life” regime where  $\sigma'_e = 0.5R_m$  (Table 3.1 page 80),  $\sigma_e = 0.7\sigma'_e$  and  $K_t = 2.5$ .

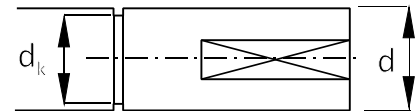


### Problem 3.8: Infinite life design drive shaft under rotary bending

Consider a grooved shaft in rotary bending. The diameter of the shaft  $d = d_1 = 20 \text{ mm}$ , the diameter of the groove  $d_k = 19 \text{ mm}$ . The stress concentration factor  $K_t = 5$ .

a) Calculate  $M_1/M_2$  where

- $M_1$  is the endurance strength for rotary bending of a 12 mm diameter shaft without groove and
- $M_2$  is the endurance strength for rotary bending of a grooved shaft of  $d=20 \text{ mm}$ ,  $d_k=19 \text{ mm}$  and  $K_t=5$ .



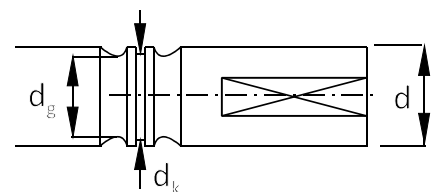
b) Calculate how much weight can be saved when the grooved shaft is replaced by an un-grooved shaft of smaller diameter  $d_2$  with the same endurance strength. Calculate the percentage [%] of weight saving  $(m_1 - m_2)/m_1$  where  $m_1$  is the mass of the grooved shaft and  $m_2$  is the mass of the ungrooved shaft.

### Problem 3.9: Infinite life design drive shaft under rotary bending

Consider a grooved shaft in rotary bending. The diameter of the shaft  $d = d_1 = 20 \text{ mm}$ , the diameter of the groove  $d_k = 19 \text{ mm}$ . The stress concentration factor  $K_{t1} = 6$ .

When stress release grooves are applied next to the sharp groove then  $K_{t2} = 2$ . The groove depth of the stress release grooves is 50% larger than the depth of the sharp groove.

How much will the bending moment that can be sustained for infinite life increase, when applying stress release grooves.



a) The grooved shaft with  $d_1=20 \text{ mm}$ ,  $d_k=19 \text{ mm}$  according ISO and  $K_{t1}=6$  is replaced by one with  $d_2=14 \text{ mm}$ ,  $d_k=13.4 \text{ mm}$  according ISO,  $d_g=13.1 \text{ mm}$  and  $K_{t2}=2$ . Calculate  $M_2/M_1$ .

b) Calculate the mass ratio  $(m_1 - m_2)/m_1$  as indication of weight saving.



### Problem 3.10: Cyclically loaded bolted structure

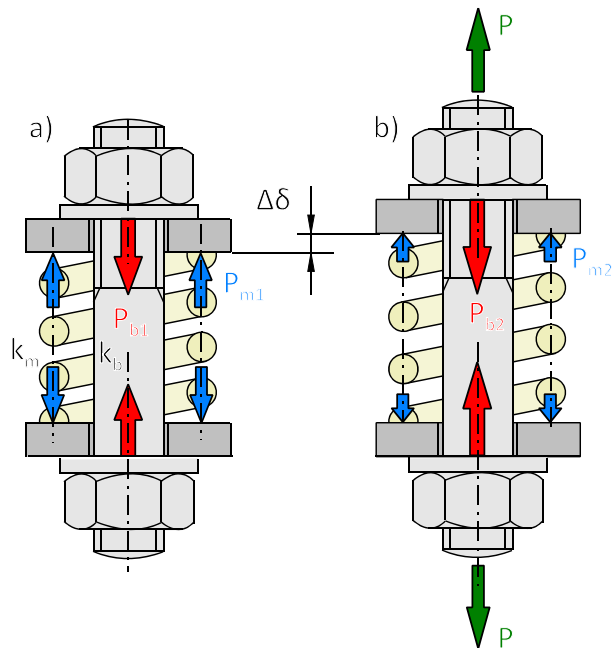
Consider a cyclically loaded bolted structure using M10-8.8 bolts. The joint stiffness  $k_m = 3k_b$  (Figure 3.30 page 91). All bolts are tightened by a preload between  $0.6F_{0.2} < F_i < 0.8F_{0.2}$ . Calculate:

- the ratio between the yield strength  $F_{0.2}$  and the fatigue strength  $P_b = 2F_a$  of the bolt.
- the maximum external load  $P$  that is limited by the bolts yield strength.
- the maximum external load  $P$  that is limited by the clamping force ( $F_m \geq 0$ ).
- the preload  $F_i(F_{0.2})$  that will result in the maximum load capacity. In this case both the yield strength and clamping force needs to be considered.
- the maximum external load  $P$  that is limited by the endurance strength of the bolts.
- Give some reasons for why the preload is defined within a range. List some more influence factors that may affect the reliability of the load capability.



### Problem 3.11: Spring model of a screw joint

Derive an equation for  $P_b/P$  expressed in  $k_b$  and  $k_m$ , where  $P_b$  is the load fluctuation in the threaded section between the nuts,  $P$  is the load fluctuation applied to the screw joint and  $k_m$ ,  $k_b$  is the stiffness of the clamped material and the bolt respectively.



### Problem 3.12: On the design of cyclically loaded screw joints

- Calculate the maximum load fluctuation  $P$  of the bolted structure as shown, where  $P$  is the maximum load fluctuation of each screw joint. Consider M24-8.8 bolts with a pretension of  $0.6R_{p0.2}$  and a joint stiffness factor  $C_m = 1/4$ . The bolt stress must remain in the endurance strength regime.
- In what way would you redesign the structure in order to obtain an improved fatigue strength?



**Problem 3.13: On the design of cyclically loaded screw joints**

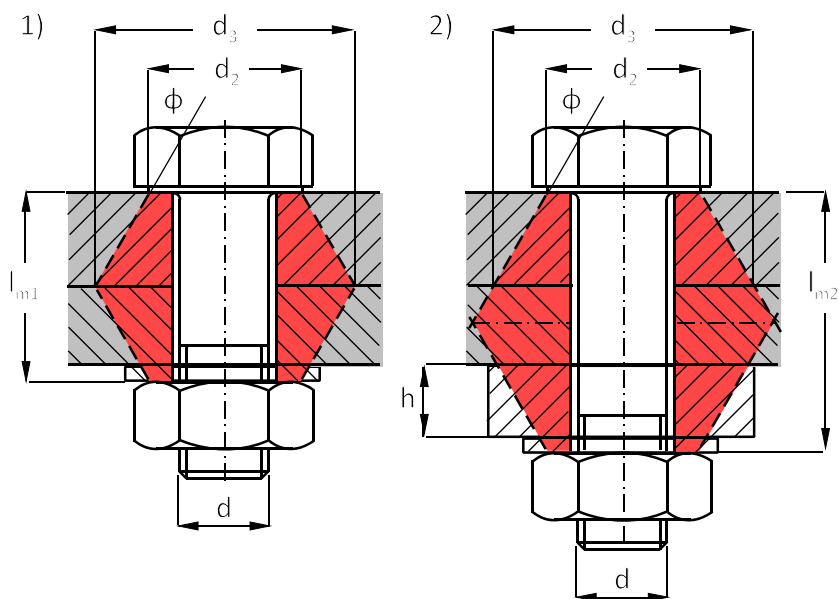
Calculate the maximum load fluctuation  $P$  of each screw joint shown in the figure. Consider M24-8.8 bolts with a pretension of  $0.8R_{p0.2}$  and a joint stiffness factor  $C_m=1/6$ .

The bolt stress must remain in the endurance strength regime, should not yield and the clamping should not be lost.

**Problem 3.14: Improved fatigue strength of a screw joint**

Calculate the factor to which the fatigue strength  $P$  of the screw joint will increase by placing a ring below the nut, where  $P$  is the maximum load fluctuation in the endurance strength regime of the bolt stress.

Consider  $l_{m1}=2d$  and  $l_{m2}=3d$ .

**Problem 3.15: Improved fatigue strength of a screw joint**

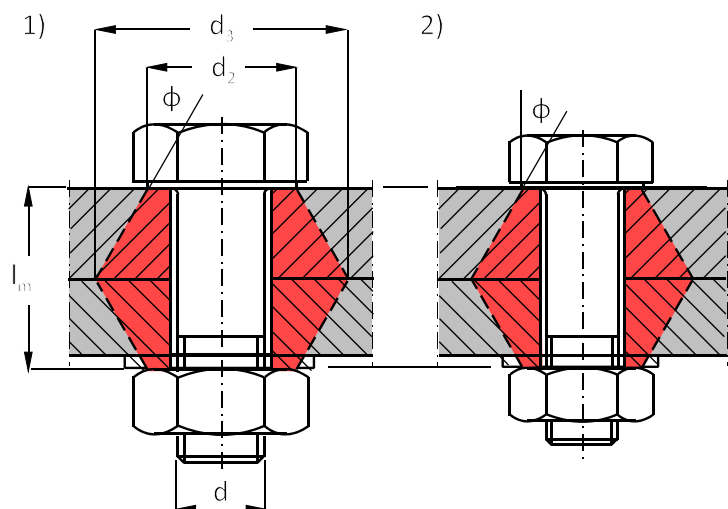
Calculate the factor to which the fatigue strength  $P$  of the screw joint will increase or decrease by replacing a metric M12-8.8 screw by a metric M10-10.9 screw. The thickness of the clamped members  $L_m=2d$  where  $d=12\text{mm}$  are kept the same.

The endurance strength of steel grade 10.9 fasteners can be calculated with

$$P_b = 2\sigma_a A_t$$

$$\sigma_a = 0.75 \left( \frac{180}{d} + 52 \right) \frac{70}{60}$$

where the factor  $70/60$  is derived from Figure 3.28 page 90 on the left.

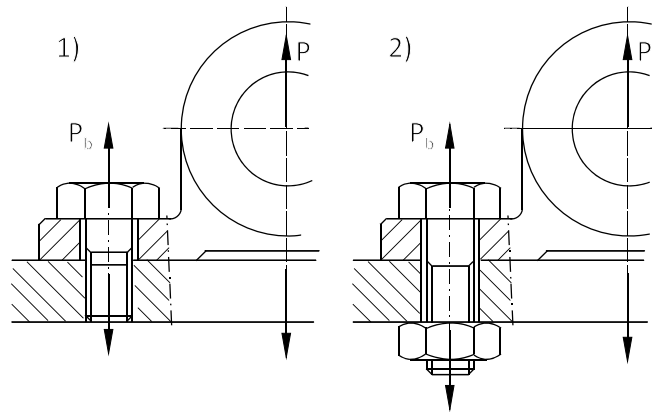




**Problem 3.16: Improved fatigue strength mounting of a bearing housing**

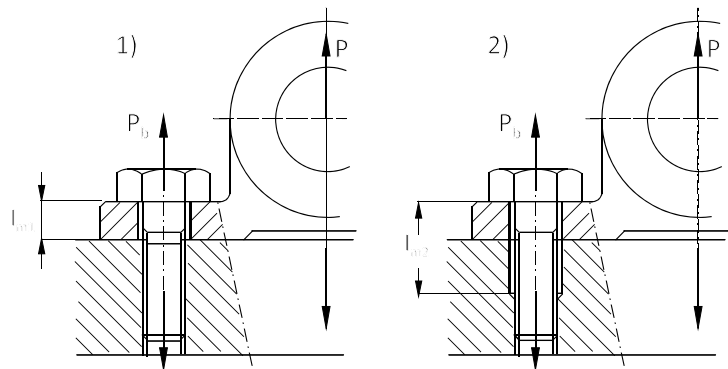
Consider two possible configurations for the mounting of a bearing housing. Both configurations are realised with M12 bolts sufficiently preloaded. The clamping length of the configurations 1 and 2 are  $L_{m,1}=0.8d$  and  $L_{m,2}=2d$ .

Calculate  $P_2/P_1$ , where  $P_1$  and  $P_2$  are the maximum load fluctuations on the bearings of configuration 1 and 2 respectively

**Problem 3.17: Improved fatigue strength mounting of a bearing housing**

Consider two possible configurations for the mounting of a bearing housing. The clamping length of the configurations 1 and 2 are  $L_{m,1}=d$  and  $L_{m,2}=2.5d$ .

Calculate  $P_2/P_1$ , where  $P_1$  and  $P_2$  are the maximum load fluctuations on the bearings of configuration 1 and 2 respectively

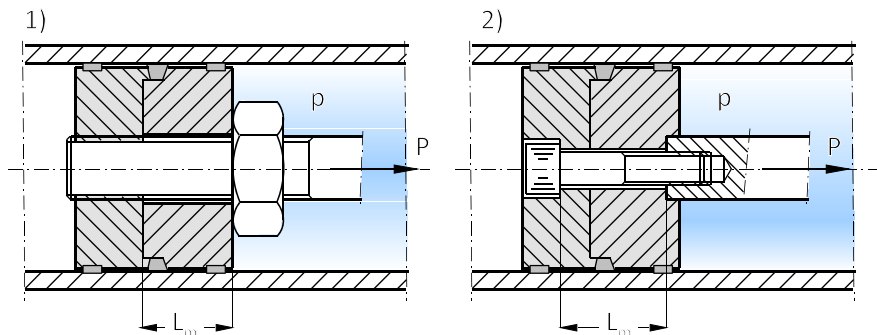
**Problem 3.18: Improved fatigue strength piston - rod connection**

Consider two possible configurations for a hydraulic piston - rod connection.

Configuration 1) M24-8.8, clamped over  $L_m=d$ , preloaded with  $0.8F_{0.2}$

Configuration 2) M12-8.8, clamped over  $L_m=3d$ , preloaded with  $0.8F_{0.2}$

Calculate the ratio of the fatigue strength  $P_2/P_1$ .  $P_1$  and  $P_2$  are the piston forces of configuration 1 and 2 respectively, that can be sustained for infinite life. The small letter  $p$  is the hydraulic pressure.

**Problem 3.19: Fatigue failure probability of a screw joint**

Consider a pneumatic cylinder that consists of an aluminium bushing with two end caps clamped by 4 steel bolt studs.

Bolt studs:  $d_b = 6$  mm,  $E_{\text{steel}} = 210$  GPa

Bushing:  $D_{\text{cyl}} = 80$  mm, wall thickness  $s = 4$  mm,  $E_{\text{alum}} = 70$  GPa

a) Calculate the joint stiffness factor  $C_m$ .

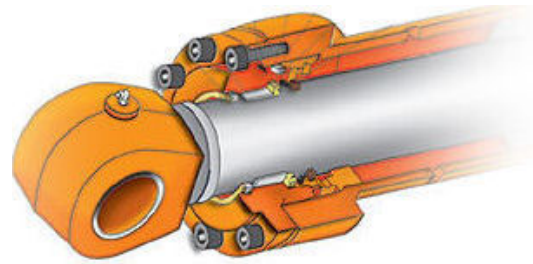
b) The bushing is replaced by one with a 2 mm wall thickness. What consequences will this deviation in joint stiffness have for the fatigue strength of the bolted connection.



### Problem 3.20: Infinite life design cylinder head screws

When more, but smaller diameter screws are applied, having the same cumulative tensile area and the same clamping force, then the fatigue strength of the structure will increase by a factor  $X$ , what is  $X$ ?

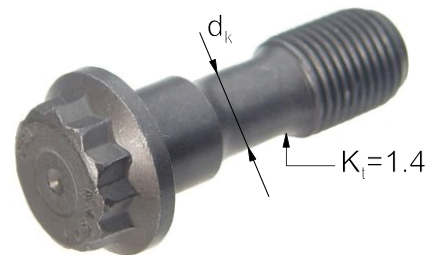
Consider twelve M24-8.8 screws to be replaced by a larger number M12-8.8 screws. The clamping length is  $L_m=20$  mm.



### Problem 3.21: Improved fatigue strength using stretch bolts

Stretch bolts are used by car manufacturers for several reasons; to accommodate LCF thermal expansion (thermo-mechanical fatigue TMF), to withstand HCF load and finally they can be prestressed accurately.

Consider a metric M12-8.8 bolt. The endurance limit of a cylindrical bar of the same steel grade loaded in axial tension is approximated by  $\sigma_e = 0.2R_m$ . The diameter of the shank is partially reduced to  $d_k = 8$  mm. The stress concentration factor in the fillets is  $K_t=1.4$ .



a) Calculate  $P_b / \Delta F$  where  $P_b$  is the endurance limit (in kN) of the threaded part and  $\Delta F$  is the endurance limit (in kN) of the in diameter reduced shank.

### Problem 3.22: Infinite life design cylinder head studs

Cylinder head stud bolts clamp the cylinder head to the block. To maintain a tight leak-free seal, the bolts must be tightened in the proper sequence to specifications.

One of the methods to torque head bolts is called torque to yield (TTY). It means that the equivalent stress is increased until it reaches the yield strength during tightening. A rule of thumb says that the torsional stress increases the equivalent stress by approximately 20% and is released as a result of relaxation after tightening.

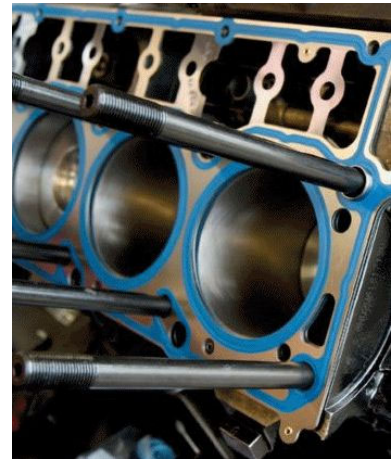


Calculate the maximum load fluctuation  $P_b$  and  $P$  that can be taken by each stud. Consider M10 threaded studs,  $A_t=58 \text{ mm}^2$ , steel grade 12.9 and  $C_m=1/8$ , an initial tensional stress of  $0.8F_{0.2}$  and a minimum clamping force that is needed to ensure leak-free sealing of  $0.2F_{0.2}$ .

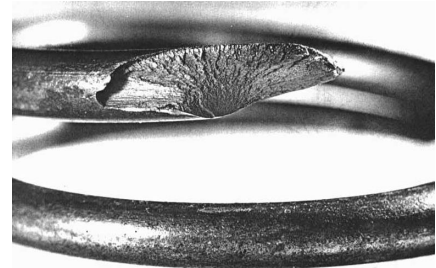
**Problem 3.23: Joint Stiffness factor**

Derive the joint Stiffness factor  $C_{m.al}$  for a clamped aluminum material by a steel bolt stud from the  $C_{m.steel}$  which is based on a clamped steel material and a steel bolt stud.

Consider  $d=10$  mm diameter bolt studs and a clamping length of  $L_m=150$  mm,  $E_{st}=210$  GPa,  $E_{al}=70$  GPa.

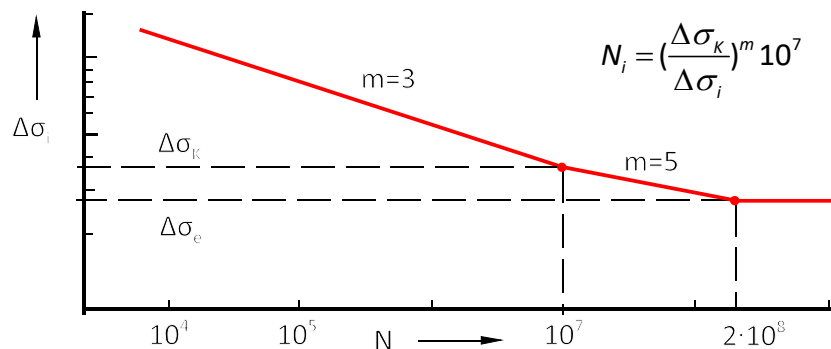
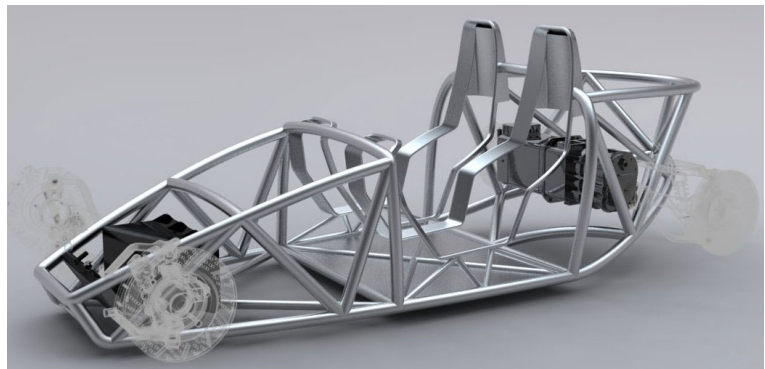
**Problem 3.24: Infinite life design compression spring**

Calculate the max amplitude of a compression spring for infinite life. Consider a wire diameter  $d=10$  mm, number of winds  $n=8$ , radius of the winds  $r=50$  mm, a shear modulus of  $G=80$  GPa,  $R_m = 2220 - 820 \log d$  where  $d$  [mm] and  $R_m$  [MPa], a fatigue strength for  $10^7$  stress cycles of  $\tau_e / R_m = 0.15$ . Approximate equations for spring stiffness of coil springs are listed in Table 14.2.26 page 450.

**Problem 3.25: Safe life design of a welded chassis**

Two hot rolled steel sections of S235 ( $R_{p0.2} = 235$  MPa) are connected by welding. The fatigue strength of the welded zone is characterised  $\sigma_K = 30$  MPa (SN-diagram shown below).

Calculate the number of stress cycles that can be sustained with stresses as high as the yield strength of the structural steel itself.



### Problem 3.26: Safe life design of butt-weld connections in pipe flanges

Socket weld pipe flanges (Fig. 1) actually slip over the pipe. These pipe flanges are typically machined with an inside diameter slightly larger than the outside diameter of the pipe. Socket pipe flanges, are secured to the pipe with a fillet weld around the top of the flange.



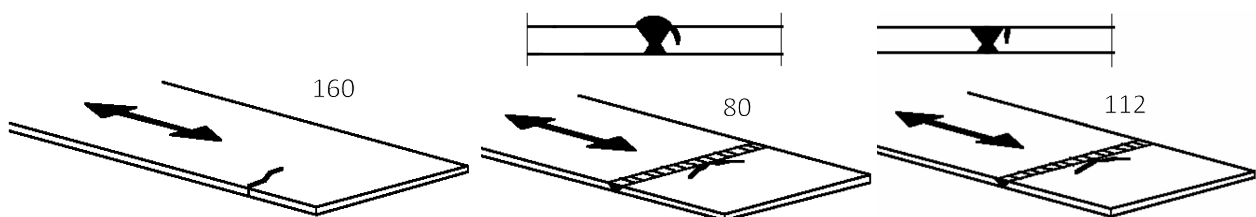
Weld neck flanges (Fig. 2) attach to the pipe by welding the pipe to the neck of the pipe flange with a butt weld. The neck allows for the transfer of stress from the weld neck pipe flanges to the pipe itself. Weld neck pipe flanges are often used for high pressure applications. The inside diameter of a weld neck pipe flange is machined to match the inside diameter of the pipe.

Calculate the ratio of the fatigue life of the weld connection in a “socket weld pipe flange” with respect to the “weld neck pipe flange”,  $N_s/N_n$ . The Socket weld flange is typically a detail category with  $\Delta\sigma_k = 30$  MPa, the weld neck flange with  $\Delta\sigma_k = 45$  MPa (SN-diagram shown in previous problem). Consider a fatigue load inducing a stress  $\Delta\sigma_i$  over the cross section of the pipe of 50 MPa.

### Problem 3.27: Safe life design of butt-weld connections

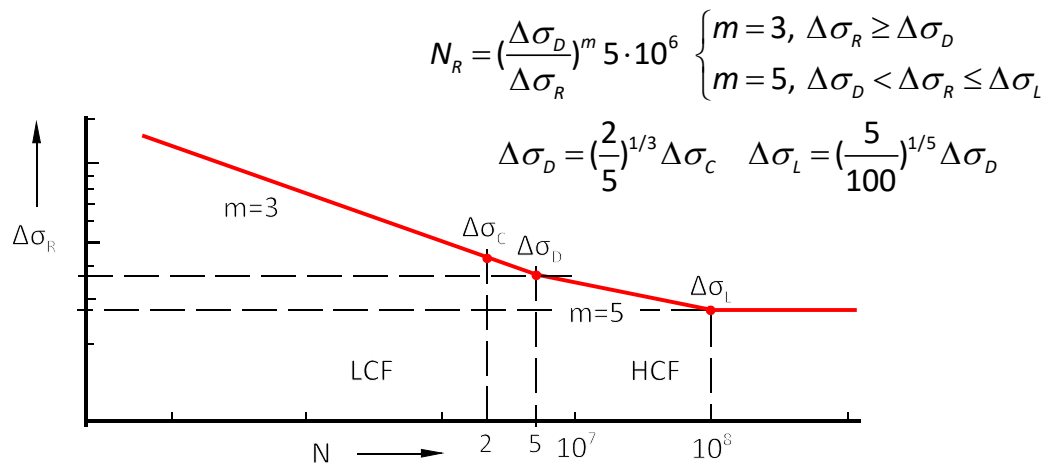
Steel grades known as S235, S275 and S355 are non-alloy structural steels. The steel grades of the JR, JO, J2 and K2 categories are in general suitable for all welding techniques. The yield strength of S235 for example is 235 MPa. The strength of statically loaded full penetrated butt welds do not need to be calculated separately, since the strength of the weld material is at least as strong as that of the structural steel.

The fatigue strength of different welds are classified by a detail category number (Eurocode 3). The detail category number indicates the reference fatigue strength  $\Delta\sigma_c$  for  $N = 2 \cdot 10^6$  load cycles. The category number 160, which indicates a fatigue strength  $\Delta\sigma_c = 160$  MPa is the highest. No detail can reach a better fatigue strength. It is the fatigue strength for unnotched sections and plates. A full penetration butt weld between two plates reduces the fatigue strength to  $\Delta\sigma_c = 80$  MPa. Finishing the butt weld by grinding improves the fatigue strength to  $\Delta\sigma_c = 112$  MPa.





The fatigue life expressed in the number of stress cycles  $N_R$  can be calculated from the actual stress in the weld  $\Delta\sigma_R$  and the fatigue strength  $\Delta\sigma_c$  according to



a) Calculate the fatigue life  $N_R$  of two strips connected by a transverse butt weld detail category 80, when cyclically loaded in tension. Consider a fully reversed stress of  $\Delta\sigma_R = 100$  MPa.

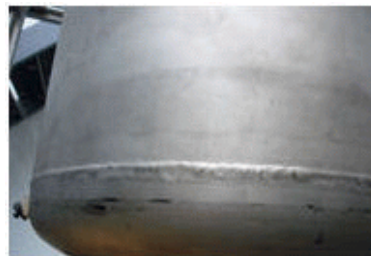
b) To what extent will the fatigue life be improved when the weld is ground flat.

### Problem 3.28: Safe life design of a butt-weld

Consider the welded cover of a pressure vessel shown in the Figure below. The butt weld is loaded in transverse direction (detail 112) and in longitudinal direction with the weld toe (detail 125). The wall thickness of the pressure vessel  $t = 5$  mm, the diameter  $D = 0.3$  m.

The pressure in the vessel fluctuates with 10 MPa. The effect of the mean pressure is left out of consideration in a first approach.

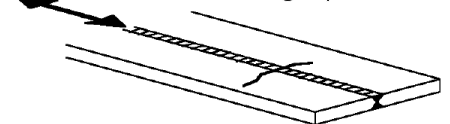
Calculate the number of times  $N$  ( $10^3$  cycles) that the vessel can be pressurized.



Detail category 112



Detail category 125



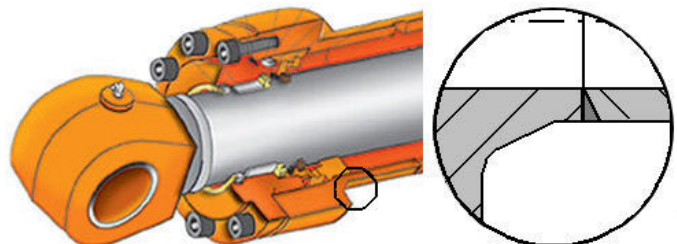
### Problem 3.29: Safe life design of butt-weld connections using the Miner's rule

Since parts are seldom stressed repeatedly at only one stress level, the cumulative damage effect of operations at various levels of stress need to be considered to estimate the fatigue life.

Calculate the fatigue life  $L$  (hr) of a full penetrated butt weld connection of Detail category 80, located between the cylinder bushing and the end cap of a hydraulic cylinder.

The actual stresses in the weld are measured during one hour of service. The stress spectrum is simplified into the values which are listed in the table below.

$\Delta\sigma_i$ [MPa]	240	120	60	35	20
$n_i$ [-]	10	20	40	400	1000



**Problem 3.30: Safe life design of a fillet-weld loaded in axial tension**

a) A fillet weld of a pipe flange connection is loaded in axial tension. Calculate the maximum load  $F$ .

Consider a pipe diameter  $D_{\text{pipe}} = 100$  mm pipe with a wall thickness  $t = 5$  mm and  $a = 3$  mm. The permissible equivalent stress is  $\sigma_{\text{eq}} = 235$  MPa.

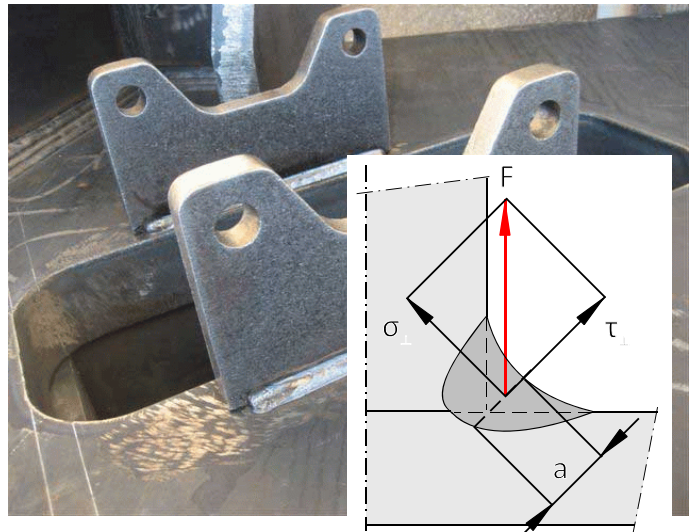
b) A fillet weld of a pipe flange connection is loaded cyclically in axial tension. Calculate the permissible equivalent stress for a fatigue life of  $1 \cdot 10^4$  load cycles. Consider a root failure of a partial penetration fillet weld of a circular structural hollow section to an end plate with detail category 40.


**Problem 3.31: Safe life design of fillet-weld connections**

A crane picks and place a load from position A to position B and back.

a) Calculate the static load  $F/(aL)$  that the fillet welds can sustain.  $L$  is the length of the weld over which the load is uniformly distributed and " $a$ " is the cross section at the throat. The yield strength of the weld material is 235 MPa.

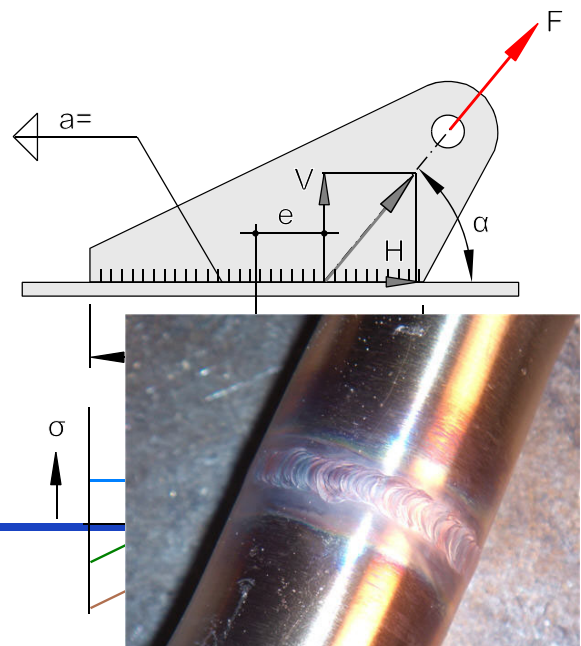
b) The stress spectrum in the welded zone of the lifting plates is simplified to  $n_1 = N/2$  times  $\Delta\sigma$  and  $n_2 = N/2$  times  $\Delta\sigma/3$  where  $N$  is the fatigue life expressed in load cycles. Consider a stress  $\Delta\sigma = 90$  MPa and a fillet weld root failure with "Detail category 40". Calculate the fatigue life  $N$ .


**Problem 3.32: Safe life design of a fillet-weld subjected to bending**

Consider the lift plate loaded in bending. Derive an equation for the maximum bending stress in the fillet weld. The lift plate is welded with a double fillet weld with a cross section of the throat  $a$ . The thickness of the lift plate is  $t$ .

a) Calculate the maximum value of the equivalent stress in the lift plate

b) Calculate the maximum value of the equivalent stress in the throat of the fillet weld.


**Problem 3.33: Eurocode 3, EN 1993-1-9 : 2003**

Two pipes are connected by a full penetration butt weld which is classified according Eurocode 3 EN 1993-1-9 by Detail category 71. The connection is cyclically loaded in bending.

Calculate the ratio between the bending moment  $M$  that can be sustained for  $N_R=10^6$  load cycles and the bending moment that can be sustained for infinite life.

### Problem 3.34: Eurocode 3, prEN 1993-1-9 : 2003

A lifting lug is welded on a steel girder. The unnotched girder is classified according Eurocode 3 by Detail category 160. By welding the lug on the steel surface of the girder, the fatigue strength of the girder has reduced to a Detail category 112.

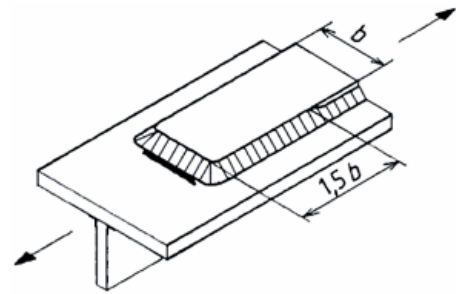


a) Calculate the ratio between the bending moment  $M$ , that can be sustained for  $N_R = 10^6$  load cycles, of a notched girder  $M_n$  and of an unnotched girder  $M_u$ .

b) Calculate the reduction in service life  $N_n/N_u$  if the load cycles are  $\Delta\sigma_R=70$  MPa.

### Problem 3.35: Eurocode 3, prEN 1993-1-9 : 2003

If a lifting lug welded directly on a steel girder will deteriorate the fatigue strength of the girder classified by detail category 80. To overcome this problem the designer has suggested to weld a cover plate on the girder at which the lifting lug is welded. What do you think, is this a good suggestion?



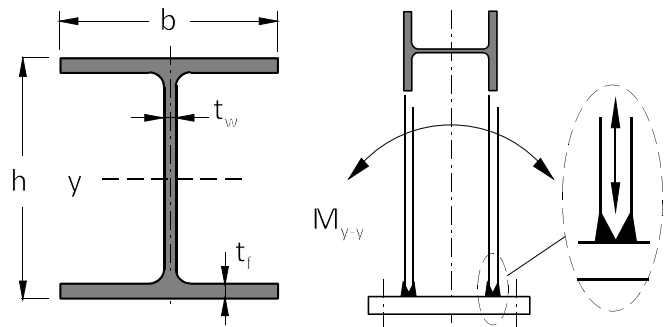
Hint: Cover plates welded on beams and plate girders are classified according Eurocode 3 by Detail category 50 and smaller, dependent on the thickness of the cover plate.

### Problem 3.36: Safe life design butt-weld loaded in bending

A butt weld between the end of a girder and a rigid end plate is cyclically loaded by the bending moment  $M_{y-y}$ . The bending moment is related to the normal stress  $M_{y-y} = W_y \sigma$  where  $W_y$  is the section modulus.

The butt weld is classified according Eurocode 3 by Detail category 45. The required service life with 95% probability is 10 years. In 10 years approximately  $10^5$  load cycles should be accommodated.

Calculate the permissible stress  $\Delta\sigma_R$ .



### Problem 3.37: Safe life design fillet-weld loaded in bending

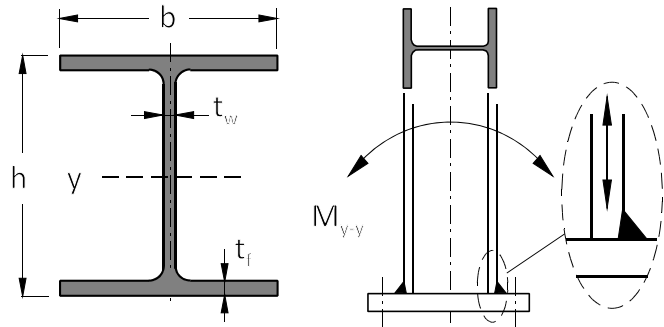
A fillet weld between the end of a girder and a rigid end plate is cyclically loaded by the bending moment  $M_{y-y}$ . The bending moment results in a tension / compression load in the welded zone.

The fillet weld is classified according Eurocode 3 by Detail category 40. The cross section of the throat sizes  $a$ , where  $a=6$  mm and  $b=100$  mm. The height of the girder  $h = 100$  mm.

The required service life with 95% probability is 10 years. In 10 years approximately  $10^5$  load cycles should be accommodated.

a) Calculate the permissible stress  $\sigma_{eq}$  in the throat of the fillet weld ( $\sigma_{eq} = \Delta\sigma_R/2$ ).

b) Calculate the permissible bending moment  $M_{y-y}$ .



### Problem 3.38: True or Untrue?

Read the related chapter of the course book before answering the true / untrue statements. These statements are useful to examine whether you understand the theory discussed in the related chapter.

1. The stress concentration factor  $K_f$  can be calculated using FEM (page 73).
2. Finite element modelling (FEM) will underestimate stress concentrations when applying a coarse grid.
3. Smooth cylindrical drive shafts without any stress concentration will not fail by fatigue (page 79).
4. It is always safe to use the geometrical stress concentration factor  $K_t$  rather than the fatigue stress concentration factor  $K_f$  while  $K_t \geq K_f$  (page 73).
5. Fatigue failure might result from cyclic loading only, if there is some tension in each stress cycle (page 73).
6. Roller burnishing introduces residual compressive stresses at the surface of a shaft or in circular grooves. The introduction of residual compressive stresses improve the fatigue strength (page 89).
7. For steel shafts under rotary bending the ratio of endurance strength  $\sigma_e$  to the 0.2% yield strength  $R_{p0.2}$  is taken  $\sigma_e/R_{p0.2} \approx 1/3$ , in a first approximation (page 80, one third of  $R_m$ )
8. The SN-curve in the SN diagram of Eurocode 3 refers to the fatigue strength with 95% reliability / 5% failure probability. (True).
9. The Smith Diagram also called Goodman Diagram provides information about the fatigue strength in the low cycle fatigue regime (page 81).
10. Fatigue failure occurs only at stresses below the yield strength (page 79)
11. For steel shafts under rotary bending the ratio of endurance strength  $\sigma_e$  to ultimate tensile strength  $R_m$  is taken  $\sigma_e/R_m \approx 1/3$ , in a first approximation (page 80).
12. When a crack is formed it creates stresses larger than those from the original notch (page 73).



13. Fatigue fractured drive shafts in torsion typically show a fracture face under 45 degrees with the cross section (page 74).
14. The HCF regime starts at  $10^7$  load cycles (page 79).
15. The Modified Goodman Diagram also called the Smith diagram provides information about the influence of the mean stress on the endurance strength (81).
16. The fatigue strength of a screw joint can be improved by making the screw more elastic with respect to the clamped material (page 92, 95).
17. The weakest cross section of a dynamically loaded fastener is at the fillet under the bolt head (page 89).
18. A screw joint with a joint stiffness factor of  $C_m=0.2$ , using M12-8.8 bolts will have a higher fatigue strength [N] than one with  $C_m=0.5$  using M24-8.8 bolts.

## Solutions

3.1)

$$a) \quad \frac{N_R}{N_K} = \left( \frac{\Delta\sigma_K}{\Delta\sigma_R} \right)^m \quad m=8, m=12$$

b)

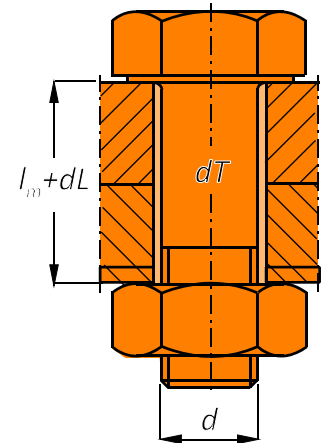
$\Delta\sigma_i$ [MPa]	240	120	60
$n_i$	10	20	40
$N_i = \left( \frac{\Delta\sigma_K}{\Delta\sigma_i} \right)^{m_i} N_K$	$m_1=8$	$m_2=12$	$m_3=12$
$D_i = \frac{n_i}{N_i}$			

The cumulative damage D during one hour of service is:  $D = \sum_{i=1}^5 \frac{n_i}{N_i} \quad L = \frac{T}{D} = 737 \text{ hr}$

c)  $D_1/D = 0.994$ , so 99.4% of the service life is consumed by the 10 cycles of the higher load.

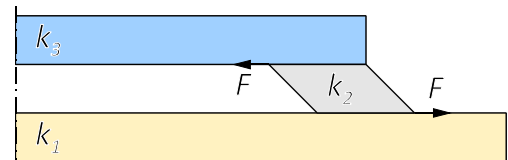
3.2)

$$\left. \begin{aligned} dL &= L_m \alpha_m dT - \frac{dF}{k_m} \\ dL &= L_m \alpha_b dT + \frac{dF}{k_b} \end{aligned} \right\} \quad dF = \frac{L_m (\alpha_m - \alpha_b) dT}{1/k_m + 1/k_b} \quad k_b \approx \frac{EA}{L_m} \quad \Delta\sigma = \frac{dF}{A_t} = 287 \text{ MPa}$$



3.3)

$$\left. \begin{aligned} u_1 &= L \alpha_1 dT_1 - \frac{F}{k_1} \\ u_3 &= L \alpha_3 dT_3 + \frac{F}{k_3} \\ u_2 &= \frac{F}{k_2} \quad u_1 = u_2 + u_3 \end{aligned} \right\} \quad F = \frac{L (\alpha_1 dT_1 - \alpha_3 dT_3)}{\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}} \quad \tau = \frac{F}{A}$$



$$k_1 = \frac{E_1 A_1}{L} \quad k_3 = \frac{E_3 A_3}{L} \quad \tau = G \arctan\left(\frac{u_1 - u_3}{h}\right) \approx G \frac{u_1 - u_3}{h} \quad \frac{F}{A} = G \frac{u_1 - u_3}{h} \quad k_2 = \frac{GA}{h}$$

This model is useful to analyse to what extent design parameters affect the shear stress.

3.4) The torque and shear stress are related according (eq. 3.8 page 84 and Table 14.2.3 page 438)

$$a) \frac{D_{dyn}}{D_{stat}} = \left( \frac{\tau}{\tau_e} \right)^{1/3} = \left( \frac{0.58 R_{p0.2}}{0.58 \sigma_e} \right)^{1/3} = \left( \frac{0.58 R_{p0.2}}{0.58 \cdot 0.5 \cdot 0.7 R_m} \right)^{1/3} = \left( \frac{0.58 \cdot 0.6 R_m}{0.58 \cdot 0.5 \cdot 0.7 R_m} \right)^{1/3} = 1.2$$

$$b) \frac{D_{dyn}}{D_{stat}} = \left( \frac{0.58 \cdot 0.6 R_m}{0.58 \cdot 0.5 \cdot 0.7 R_m / 1.7} \right)^{1/3} = 1.43$$

3.5) a) Statically loaded

$$T = \frac{\pi}{16} D^3 \left( 1 - 1.7 \frac{d}{D} \right)^3 \tau, \quad D = \left( \frac{T}{\frac{\pi}{16} \left( 1 - 1.7 \frac{d}{D} \right)^3 \tau} \right)^{1/3} = 10.1 \text{ mm}$$

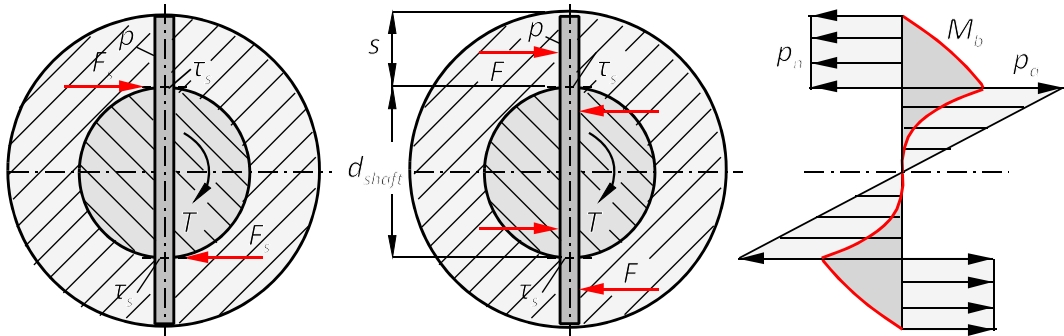
b) Dynamically loaded

$$T = \frac{\pi}{16} D^3 \left( 1 - 1.7 \frac{d}{D} \right)^3 \frac{\tau_e}{K_t}, \quad \tau_e = 0.58 \cdot 0.7 \cdot 0.5 \cdot R_m, \quad K_t = 2.034, \quad D = 15.3 \text{ mm}$$

c)

$$T = F_{shear} D, \quad \tau_{pin} = \frac{F_{shear}}{\frac{\pi}{4} d^2}, \quad \frac{\tau_{shaft}}{\tau_{pin}} = \frac{\frac{\pi}{4} d^2 D}{\frac{\pi}{16} D^3 \left( 1 - 1.7 \frac{d}{D} \right)^3} = \frac{4 \left( \frac{d}{D} \right)^2}{\left( 1 - 1.7 \frac{d}{D} \right)^3} = 0.557$$

3.6)



Bending

$$T = F(d_{shaft} + s), \quad M_b = F \frac{s}{2}, \quad M_b = \frac{s}{2(d_{shaft} + s)} T, \quad s = \frac{d_{hub} - d_{shaft}}{2}$$

$$M_b = \frac{\pi}{32} d_{pin}^3 \sigma_b, \quad \sigma_b = \frac{32}{\pi d_{pin}^3} \frac{s}{2(d_{shaft} + s)} T$$

Shear

$$T = F_s d_{shaft}, \quad \tau = \frac{F_s}{\pi/4 d_{pin}^2} = \frac{T}{\pi/4 d_{pin}^2 d_{shaft}}$$

Ratio of stresses

$$\frac{\sigma_b}{\tau} = \frac{32}{\pi d_{pin}^3} \frac{s \pi/4 d_{pin}^2 d_{shaft}}{2(d_{shaft} + s)} = 4 \frac{s/d_{pin}}{(1 + s/d_{shaft})} = 4 \left( \frac{d_{hub}}{d_{shaft}} - 1 \right) \left( \frac{d_{pin}}{d_{shaft}} \left( 1 + \frac{d_{hub}}{d_{shaft}} \right) \right)^{-1} = 5.7$$

Pinned joints using hardened steel pins in general fail by fracture from bending. Splitter pins also known in the US as cotter pin are typically made of low carbon steel and their failure mechanism would be shear failure.

## 3.7) a) Statically loaded

$$M = \frac{\pi}{32} D^3 \sigma, \quad D = \left( \frac{M}{(\pi/32) R_{p0.2}} \right)^{1/3} = 5.1 \text{ mm}$$

## b) Dynamically loaded

$$M = \frac{\pi}{32} D^3 \sigma, \quad D = \left( \frac{M}{(\pi/32) 0.70.5 R_m / K_t} \right)^{1/3} = 8.3 \text{ mm}$$

$$3.8) \text{ a) } M_1(d=12) = \frac{\pi}{32} d^3 \sigma_e, \quad M_2(d_k=19) = \frac{\pi}{32} d_k^3 \frac{\sigma_e}{K_t}, \quad \frac{M_1(d=12)}{M_2(d_k=19)} = \frac{d^3}{d_k^3 / K_t} = 1.26$$

$$\text{b) } M = \frac{\pi}{32} d_2^3 \sigma_e, \quad M = \frac{\pi}{32} d_k^3 \frac{\sigma_e}{K_t}, \quad d_2 = \frac{d_k}{K_t^{1/3}} = 11.1 \text{ mm}, \quad \frac{m_2}{m_1} = \left( \frac{d_2}{d_1} \right)^2 = 0.31 \quad \text{weight saving 69\%}$$

$$3.9) \text{ a) } d_1=20 \text{ mm}, \quad d_k=19 \text{ mm}, \quad K_{t1}=6, \quad d_g=18.5 \text{ mm}, \quad d_2=14 \text{ mm}, \quad d_g=13.1 \text{ mm}, \quad K_{t2}=2$$

$$M_1 = \frac{\pi}{32} d_k^3 \frac{\sigma_e}{K_{t1}}, \quad M_2 = \frac{\pi}{32} d_g^3 \frac{\sigma_e}{K_{t2}}, \quad \frac{M_2}{M_1} = \left( \frac{d_g}{d_k} \right)^3 \frac{K_{t1}}{K_{t2}} = 0.98$$

$$\text{b) } \frac{m_1 - m_2}{m_1} = \frac{d_1^2 - d_2^2}{d_1^2} = 0.51$$

Conclusion: 98% of the same endurance and 51% weight saving.

## 3.10) a) The stress increase in the bolt is limited to the yield strength of the bolt.

$$F_{0.2} = R_{p0.2} A_t = 37 \text{ kN}, \quad F_a = 0.75(180/d+52) A_t, \quad P_b = 2F_a = 6.1 \text{ kN}, \quad P_b / F_{0.2} = 16.4\%$$

## b) The stress increase in the bolt is limited to the yield strength of the bolt.

$$P_b = 0.2 F_{0.2}, \quad P_m = 3P_b, \quad P = P_b + P_m = 4P_b = 0.8 F_{0.2} = 29.7 \text{ kN}$$

c) The stress increase in the bolt is limited by the minimum value of the clamping force  $F_m$ . If the bolt stress is increased further the clamped members become separated.

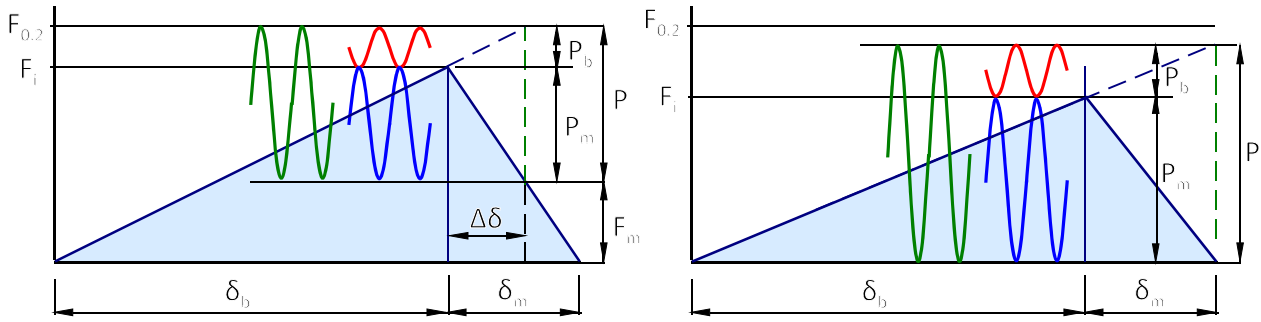
$$P_m = F_i = 0.6 F_{0.2}, \quad P_b = P_m / 3, \quad P = P_b + P_m = 0.8 F_{0.2} = 29.7 \text{ kN}$$

d) The maximum load capacity is obtained with  $P_b + P_m = F_{0.2}$ . With  $P_b : P_m = k_b : k_m$  this results in

$$F_i = (1 - C_m) F_{0.2} = 0.75 F_{0.2} = 27.84 \text{ kN}, \quad P = F_{0.2} = 37.12 \text{ kN}$$

e) When cyclically loaded, the fluctuation of bolt stress that can be sustained might be limited by the endurance strength of the bolt. The endurance strength of the M10-8.8 bolt is  $\sigma_a = 52.5 \text{ MPa}$  (Figure 3.28 page 90). This results in

$$P_b = 2 \sigma_a A_t, \quad P_m = 3P_b, \quad P = P_b + P_m = 4P_b = 24.4 \text{ kN}$$



f) Variation on the preload is mainly caused by variation on the coefficient of friction during tightening. Other important influence factors are the accuracy of the tightening method and stress relaxation after tightening.

**3.11)** The physical model of the screw joint can be expressed as two springs connected in parallel.

$$\left. \begin{aligned} P_b &= \Delta\delta k_b \\ P_m &= -\Delta\delta k_m \\ P &= P_b - P_m \end{aligned} \right\} \begin{aligned} P &= \Delta\delta (k_b + k_m) \\ \Delta\delta &= \frac{P}{k_b + k_m} \end{aligned} \quad P_b = \frac{k_b}{k_b + k_m} P = C_m P$$

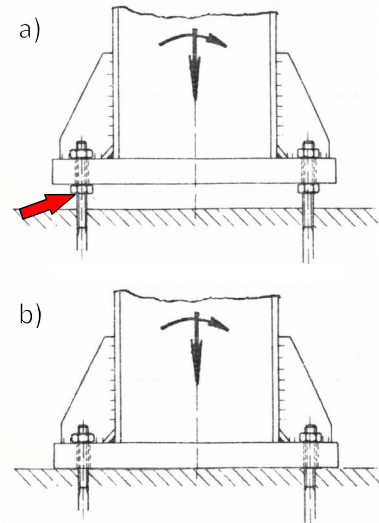
**3.12)** The critical section of the screw joint is the part below the lower nut, not the prestressed section. The fatigue strength of the critical section is:

$$\left. \begin{aligned} P &= P_b = 2\sigma_a A_t \\ \sigma_a &= 0.75 \left( \frac{180}{d} + 52 \right) \end{aligned} \right\} P = 31.5 \text{ kN}$$

A very small joint stiffness factor  $C_m$  can be obtained when using bolt studs anchored at a relative large depth (Figure b).

**3.13)** The fatigue strength of the screw joint is:

$$\left. \begin{aligned} P_b &= 2\sigma_a A_t \quad \sigma_a = 0.75 \left( \frac{180}{d} + 52 \right) \\ P &= P_b / C_m \quad C_m = 1/6 \end{aligned} \right\} \begin{aligned} P &= 189 \text{ kN} \\ (P_b < 0.2 F_{0.2}, P - P_b > F_m) \end{aligned}$$



**3.14)** The load fluctuation  $P_b$  that can be taken by the bolt is the same for both configurations, since the steel grade and diameter of the bolts are the same. The load fluctuation that can be taken by the joint  $P = P_b / C_m$  where  $C_m$  is the joint stiffness factor, it is the partition of the load  $P$  that is taken by the bolt. The factor to which the fatigue strength of the joint will increase is  $P_2 / P_1 = C_{m1} / C_{m2}$ .

Substitution of  $l_m/d=2$  in (3.12) page 92 gives  $C_{m1} = 0.232$ . Substitution of  $l_m/d=3$  gives  $C_{m2} = 0.178$ .

The fatigue strength of the screw joint will increase by a factor  $P_2 / P_1 = C_{m1} / C_{m2} = 0.232 / 0.178 = 1.3$ . In other words, the external load that can be taken by the screw joint has increased by 30%.

3.15) The static strength of the bolt has decreased by a factor  $\frac{(R_{p0.2} A_t)_2}{(R_{p0.2} A_t)_1} = \frac{0.9 \ 1000 \ 58}{0.8 \ 800 \ 84.3} = 0.97$

The fatigue strength of the bolt has decreased by a factor  $\frac{(\sigma_a A_t)_2}{(\sigma_a A_t)_1} = \frac{52.5 \ 70/60 \ 58}{50.3 \ 84.3} = 0.84$

where  $\sigma_a$  is calculated from (Figure 3.18-right page 90). The correction for the higher steel grade is 70/60 (Figure 3.90-left page 90). The tensile stress area is listed in Table 14.2.11 page 443.

The joint stiffness factor has decreased by a factor  $\frac{C_{m2}}{C_{m1}} = \frac{0.208}{0.232} = 0.90$

The static strength of the screw joint will increase by a factor  $0.97/0.90=1.08$  (8%).

The fatigue strength of the screw joint will decrease by a factor  $0.84/0.90=0.95$  (-5%).

3.16) Both mountings use the same screws:  $P_b = C_{m1} P_1 = C_{m2} P_2$   $\frac{P_2}{P_1} = \frac{C_{m1}}{C_{m2}} = \frac{0.334}{0.232} = 1.44$

3.17) Both mountings use the same screws:  $P_b = C_{m1} P_1 = C_{m2} P_2$   $\frac{P_2}{P_1} = \frac{C_{m1}}{C_{m2}} = \frac{0.334}{0.232} = 1.44$

3.18) The critical section of the screw joint in configuration 1 is the threaded part outside the clamped area, not the prestressed section. The fatigue strength of the critical area of the M24 is:

$$C_m = 1 \quad P_b = 2\sigma_e A_t \quad P_1 = P_b / C_m = 31.5 \text{ kN}$$

The joint stiffness factor and the maximum load of the M12 screw joint in configuration 2 is:

$$C_m = 0.178 \quad P_b = 2\sigma_e A_t = 8.5 \text{ kN} \quad P_2 = P_b / C_m = 47.5 \text{ kN} \quad P_2 / P_1 = 1.51$$

3.19)

$$\sigma = E \epsilon \quad \frac{F}{A} = E \frac{dL}{L} \quad k = \frac{F}{dL} = \frac{AE}{L}$$

$$\frac{k_b}{k_m} = \frac{A_b E_b}{A_{st} E_{st}} = \frac{4 \frac{\pi}{4} d^2 E_{steel}}{\frac{\pi}{4} (D^2 - (D-2s)^2) E_{alum}} \quad C_m = \frac{k_b}{k_b + k_m} = 0.262$$

The smaller the joint stiffness factor  $C_m$ , the smaller the part of external loading that is taken by the bolts. If the joint stiffness factor is larger than expected, it is the case with a small wall thickness of the bushing, the fatigue strength of the bolted connection will be smaller than expected and the bolts may fail by fatigue prematurely.

3.20) Same static strength  $\frac{(n A_t)_2}{(12 A_t)_1} = \frac{n \ 84.3}{12 \ 353} = 1 \quad n = 50$

Fatigue strength  $P_{tot} = n P \quad P = P_b / C_m \quad P_b = 2 \sigma_a A_t$

$$\frac{(P_{tot})_2}{(P_{tot})_1} = \frac{(n \sigma_a A_t / C_m)_2}{(12 \sigma_a A_t / C_m)_1} = 1.43$$

- 3.21)** The endurance limit of a metric M12-8.8 bolt is calculated from Figure 3.28 page 90,  $\sigma_a = 50$  MPa. Multiplied by the tensile stress area  $A_t = 84.26 \text{ mm}^2$  gives  $F_a = 4.2 \text{ kN}$  and  $P_b = 2F_a = 8.43 \text{ kN}$ .

The endurance limit of the steel bar in axial tension is  $\sigma_e = 0.2R_m = 160$  MPa.

The endurance strength of the shank with reduced diameter  $\Delta F = 2 \sigma_e (\pi/4) d_k^2 / K_t = 11.49 \text{ kN}$

$P_b/\Delta F = 0.734$  (the threaded part remains the most critical).

- 3.22)** The maximum load fluctuation that can be taken by each stud can be limited by:

- 1) yielding of the bolt studs when exceeding the elastic limit

The load fluctuation that can be taken by each bolt stud

$$P_b = 0.2 A_t R_{p0.2} = 0.2 F_{0.2} = 12.5 \text{ kN (Figure 3.30 page 91)}, P = P_b/C_m = 100 \text{ kN}$$

- 2) a minimum clamping force that is needed to ensure leak-free sealing and

When 20% of the yield strength of the bolt studs is needed as minimum clamping force then 60% of the preload remains for  $P_m$ , i.e.  $P_m = 0.6 F_{0.2}$

$$P_b = C_m P, P_m = (1 - C_m)P, \text{ then } P_b = (C_m/(1 - C_m)) P_m = 5.4 \text{ kN}, P = P_b/C_m = 43 \text{ kN}$$

- 3) the endurance strength of the bolt studs.

The endurance strength of the bolt studs is  $P_b = 2 \sigma_a A_t = 6.1 \text{ kN}$ ,  $P = P_b/C_m = 49 \text{ kN}$ .

Conclusion: The clamping force of the screw joint is most critical,  $P_b = 5.4 \text{ kN}$ ,  $P = P_b/C_m = 43 \text{ kN}$ .

Bolt studs are always mounted using a solid lubricant (page 258) for several reasons. First of all to ensure a constant and low friction which is profitable to accurately apply the preload by tightening and to limit torsion stresses during tightening. Secondly, the solid lubricant prevents fretting corrosion which ensures that the bolted joint can be unscrewed, and, not less important that this avoids fatigue corrosion. Especially high strength steel alloys are very susceptible to stress corrosion.

- 3.23)**

$$C_m = \frac{k_b}{k_m + k_b} = \frac{1}{(1.5 + 0.289 L_m/d)^2} = 0.029 \quad \frac{k_m}{k_b} = \frac{1}{C_m} - 1 = 33 \text{ for steel}$$

$$\text{for al. } \frac{k_{m.al}}{k_b} = 11, C_{m.al} = \frac{1}{k_{m.al}/k_b + 1} = 0.083$$

- 3.24)**

$$\left. \begin{aligned} f &= \frac{64n r^3 F}{d^4 G}, \quad F = \frac{\pi}{16} \frac{d^3}{r} \tau: \quad f = 4\pi n \frac{r^2 \tau}{d G} \\ \tau_e &= 0.15 R_m \quad R_m = 2220 - 820 \log d \end{aligned} \right\} \begin{aligned} R_m &= 1400 \text{ MPa} \quad \tau_e = 210 \text{ MPa} \quad f_{\max} = 66 \text{ mm} \end{aligned}$$

- 3.25)** Substitution of  $\sigma_{HCF} = 30$  MPa,  $\sigma_i = 235$  MPa,  $m=3$  in  $N_i = \left(\frac{\sigma_{HCF}}{\sigma_i}\right)^m 10^7$  gives  $N_i = 20.8 \cdot 10^3$  cycles

- 3.26)** The fatigue life expressed in the number of stress cycles  $N_i$  is calculated with (3.13) page 96.  $N_s/N_n = (30/45)^3 = 0.296$ . The exponent  $m=3$  for both flanges since  $\Delta\sigma_i > \Delta\sigma_k$ . Note that the fatigue life of the socked flange is only 30% of the fatigue life of the neck flange.

- 3.27)** Substitution of  $\Delta\sigma_c = 80$  MPa,  $\Delta\sigma_R = 100$  MPa and  $m=3$  in the curve fit equation of the SN-curve results in  $\Delta\sigma_D = 59$  MPa and a fatigue life of  $N_R = 1.02$  million load cycles.

When ground flat then  $\Delta\sigma_c = 112$  MPa and the fatigue life is improved by a factor  $(112/80)^3 = 2.74$ .

- 3.28)** The stress in the circumference of the pressure vessel is twice as large as the stress in the length direction. The stress in longitudinal direction with the weld toe is most critical (Detail 125). This stress is calculated with  $\Delta\sigma_R = \Delta p D / (2 t) = 300$  MPa. Substitution of  $\Delta\sigma_c = 125$  MPa and  $m=3$  in the curve fit equation of the SN-curve results in  $\Delta\sigma_D = 92$  MPa and a fatigue life of  $N_R = 145 \cdot 10^3$  load cycles.

- 3.29)** First step is to calculate  $\Delta\sigma_D$  and  $\Delta\sigma_L$  in order to find out whether the initial stresses are in the LCF regime where  $m = 3$ , in the HCF-regime where  $m=5$  or below the cut-off limit. Stresses below the cut-off limit are left out of consideration.

$$\text{Detail 80: } \Delta\sigma_c = 80 \text{ MPa} \quad \Delta\sigma_D = \left(\frac{2}{5}\right)^{1/3} \Delta\sigma_c = 59 \text{ MPa} \quad \Delta\sigma_L = \left(\frac{5}{100}\right)^{1/5} \Delta\sigma_D = 32 \text{ MPa}$$

Next step is to calculate the damage fraction  $D$  for one hour of service.  $N_i$  is the number of load cycles  $\Delta\sigma_i$  that could be accumulated until fraction.

$\Delta\sigma_i$ [MPa]	240	120	60	35	20
$n_i$ [-]	10	20	40	400	1000
$N_i$ [-]	$\left(\frac{\Delta\sigma_D}{240}\right)^3 \cdot 5 \cdot 10^6$	$\left(\frac{\Delta\sigma_D}{120}\right)^3 \cdot 5 \cdot 10^6$	$\left(\frac{\Delta\sigma_D}{60}\right)^3 \cdot 5 \cdot 10^6$	$\left(\frac{\Delta\sigma_D}{35}\right)^5 \cdot 5 \cdot 10^6$	$\infty$
$D_i$ [-]	$n_i/N_i$				

The cumulative damage  $D$  during one hour of service ( $T=1$  hr) is:

$$D = \sum_{i=1}^5 \frac{n_i}{N_i} \quad L = \frac{T}{D} = 5477 \text{ hr}$$

Note: If in one hour  $D = 0.1$ , then 10% of the service life has passed and the service life would be  $L = T/D = 1/0.1 = 10$  hours.

- 3.30)** a)  $\sigma_{eq} = \frac{F\sqrt{2}}{aL} \quad F = \frac{aL}{\sqrt{2}} \sigma_{eq} = 157 \text{ kN}$  b)  $\Delta\sigma_i = \left(\frac{2 \cdot 10^6}{110^4}\right)^{1/3} \Delta\sigma_c = 234 \text{ MPa}$

Conclusion: The static strength with  $R_{p0.2}=235$  MPa appears approximately the same as the fatigue strength  $\Delta\sigma_R$  for  $N_R = 1 \cdot 10^4$  load cycles.

**3.31)**

- a) The load is distributed over the length  $L$ :

$$\left. \begin{aligned} \sigma_{eq} &= \sqrt{\sigma_{\perp}^2 + 3(\tau_{\perp}^2 + \tau_{//}^2)} \leq R_{p0.2} \\ \sigma_{\perp} &= \tau_{\perp} = \frac{F/\sqrt{2}}{aL} \quad \tau_{//} = 0 \end{aligned} \right\} \quad \sigma_{eq} = R_{p0.2} = \frac{F\sqrt{2}}{aL} \quad \frac{F}{aL} = \frac{R_{p0.2}}{\sqrt{2}}$$

- b)  $\Delta\sigma_c = 40$  MPa,  $\Delta\sigma_D = 30$  MPa,  $m=3$  for both stress levels.



$$D = \frac{n_1}{\left(\frac{40}{90}\right)^3 2 \cdot 10^6} + \frac{n_2}{\left(\frac{40}{90/3}\right)^3 2 \cdot 10^6} = 1 \quad \left. \begin{array}{l} n_1 = n_2 \\ N = n_1 + n_2 \end{array} \right\} N = 3.4 \cdot 10^5 \text{ cycles}$$

3.32) The maximum equivalent stress in the lift plate:

$$H = F \cos(\alpha) \quad V = F \sin(\alpha) \quad M = V e$$

$$\tau_{//}(H) = \frac{H}{tL} \quad \sigma_{\perp}(V) = \tau_{\perp}(V) = \frac{V}{tL} \quad \sigma_{\perp}(M) = \tau_{\perp}(M) = \frac{M}{\frac{1}{6} t L^2}$$

$$\tau_{//} = \tau_{//}(H) \quad \sigma_{\perp} = \sigma_{\perp}(V) + \sigma_{\perp}(M) \quad \tau_{\perp} = \tau_{\perp}(V) + \tau_{\perp}(M)$$

$$\sigma_{eq} = \sqrt{\sigma_{\perp}^2 + 3(\tau_{\perp}^2 + \tau_{//}^2)}$$

The maximum equivalent stress in the fillet weld:

$$\frac{\sigma_{eq,plate}}{t} = \frac{\sigma_{eq,throat} \sqrt{2}}{2a}$$

The factor 2 in the denominator refers to the double fillet weld. The  $\sqrt{2}$  in the numerator is from  $\sigma_{eq}$  in the cross section of the throat of the fillet weld.

3.33)

$$\Delta\sigma_c = 71 \text{ MPa} \quad \Delta\sigma_D = \left(\frac{2}{5}\right)^{1/3} \Delta\sigma_c = 52 \text{ MPa} \quad \Delta\sigma_L = \left(\frac{5}{100}\right)^{1/5} \Delta\sigma_D = 29 \text{ MPa}$$

$$\Delta\sigma_R = \Delta\sigma_D / (N/5 \cdot 10^6)^{1/3} = 89 \text{ MPa} \quad \Delta\sigma_R / \Delta\sigma_L = 3.1$$

The stresses that are permitted in a safe life design for  $N_R = 10^6$  load cycles are 3.1 times higher than the stresses permitted in an infinite life design.

3.34) a) The tensile stress  $\Delta\sigma_R$  is proportional to the bending moment M.

$$\frac{M_n}{M_u} = \frac{\Delta\sigma_{R,n}}{\Delta\sigma_{R,u}} \quad \text{and} \quad \frac{\Delta\sigma_{R,n}(N_R=N)}{\Delta\sigma_{R,u}(N_R=N)} = \frac{\Delta\sigma_{C,n}}{\Delta\sigma_{C,u}} = \frac{112}{160}$$

b) Load cycles of  $\Delta\sigma_R = 70 \text{ MPa}$ . First step is to find out whether  $\Delta\sigma_D < \Delta\sigma_R < \Delta\sigma_L$  to estimate m.

$$1) \Delta\sigma_D = \left(\frac{2}{5}\right)^{1/3} \Delta\sigma_c = 117.9 \text{ MPa}, \quad \Delta\sigma_L = \left(\frac{5}{100}\right)^{1/5} \Delta\sigma_D = 64.8 \text{ MPa}, \quad N_{R1} = \left(\frac{\Delta\sigma_D}{\Delta\sigma_R}\right)^5 N_D = 6.8 \cdot 10^7 \text{ cycles}$$

$$2) \Delta\sigma_D = \left(\frac{2}{5}\right)^{1/3} \Delta\sigma_c = 82.5 \text{ MPa}, \quad \Delta\sigma_L = \left(\frac{5}{100}\right)^{1/5} \Delta\sigma_D = 45.4 \text{ MPa}, \quad N_{R2} = \left(\frac{\Delta\sigma_D}{\Delta\sigma_R}\right)^3 N_D = 1.14 \cdot 10^7 \text{ cycles}$$

$$N_{R2}/N_{R1} = 0.17$$

- 3.35) Detail category 160: Unnotched beam  
 Detail category 80: Transverse attachment like the lifting lug welded on a steel girder.  
 Detail category 50: Cover plates in beams and plate girders  
 Conclusion, the cover plate makes the situation worse.

3.36)  $\Delta\sigma_c = 45 \text{ MPa}$   $\Delta\sigma_R = \Delta\sigma_c \left(\frac{2}{0.1}\right)^{1/3} = 166 \text{ MPa}$

3.37)  $\Delta\sigma_c = 40 \text{ MPa}$   $\Delta\sigma_D = \left(\frac{2}{5}\right)^{1/3} \Delta\sigma_c = 29 \text{ MPa}$   $\Delta\sigma_R = \Delta\sigma_D \left(\frac{5}{0.1}\right)^{1/3}$   $\Delta\sigma_R = 109 \text{ MPa}$   $\sigma_{eq} = \Delta\sigma_R / 2$

$$M = F h \quad \sigma_{eq} = \frac{F\sqrt{2}}{a b} \quad M = a b h \frac{\sigma_{eq}}{\sqrt{2}}$$

- 3.38) True / Untrue

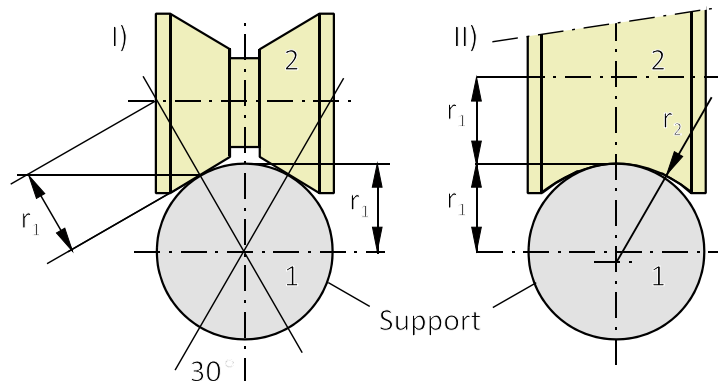
True 2, 4, 5, 6, 8, 11, 12, 13, 15, 16 Untrue 1, 3, 7, 9, 10, 14, 17, 18

## Chapter 4 - Problems with Solutions

## Problem 4.1: Effective contact radius cam-roller system

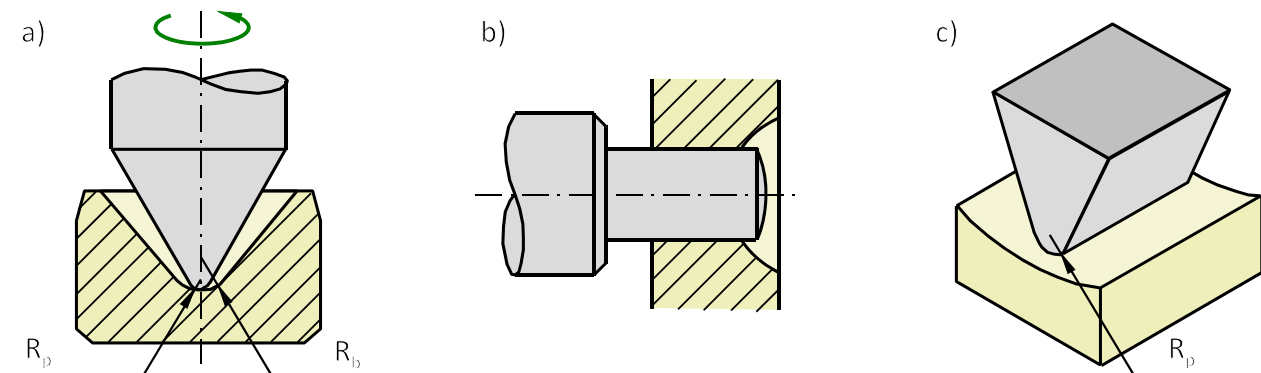
a) Calculate the effective contact radius  $R'$  of the linear cam roller guides shown in the figure. Consider a radius of the supporting bar of  $r_1=5\text{mm}$  and a curved cam with  $r_2=1.1r_1$ .

b) Calculate the static load rating  $C_0$  of the point contact illustrated by configuration I according ISO 76: 1987 (page 105). The cam and support are made of ball bearing steel 100Cr6,  $E=213\text{ GPa}$ ,  $\nu=0.29$ ,  $p_{\max}=4.2\text{ GPa}$ , radius  $r_1=5\text{ mm}$ .



## Problem 4.2: Effective contact radius knife edge jewel bearing

Jewel bearings are known from mechanical watches. Today, jewel bearings are used widely in sensitive measurement equipment and aircraft instruments. The most common types of instrument jewel bearings are a) the V bearing, b) the ring bearing and c) the knife edge bearing.



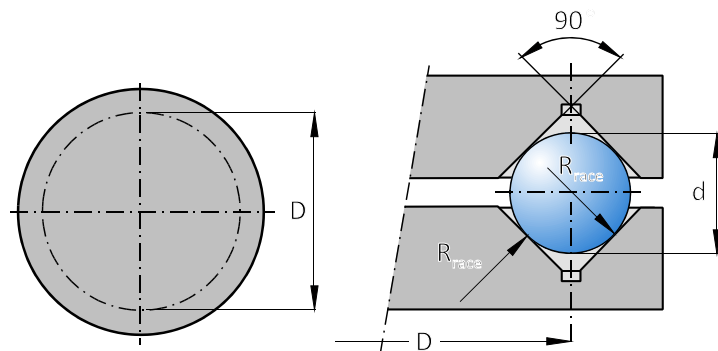
Calculate the effective contact radius  $R'$  expressed in the pivot radius  $R_p$  and bearing radius  $R_b$  of

- the pivot bearing
- the knife edge bearing

## Problem 4.3: Effective contact radius thrust bearing

A thrust bearing is designed with a V-groove in the top and bottom plate. The groove is filled with balls of diameter  $d$ .

Consider: ball diameter  $d = 4\text{ mm}$ , raceway diameter  $D = 50\text{ mm}$ , top angle of the groove  $90^\circ$ . With  $D$  relatively large compared to  $d$ , the inner and outer contact radius of the raceway in rolling direction is  $R_{\text{race}} \approx R\sqrt{2}$  where  $R=D/2$ .



Calculate the effective contact radius between the ball and the inner radius of the raceway  $R'_{\text{inner}}$  and between the ball and the outer diameter of the raceway  $R'_{\text{outer}}$ .

#### Problem 4.4: Effective contact radius thrust bearing

The number of balls that will fit in the groove of the thrust bearing described in previous problem is  $n = \text{integer}(\pi D/d)$  where  $D$  is the diameter of the groove and  $d$  the diameter of the balls. It is to be investigated what the effect is of the diameter of the balls on the load capacity of the thrust bearing. To this end, two bearings with ball diameter  $d_1$  and  $d_2$  respectively are compared.

Consider  $d_1 = 4$  mm,  $d_2 = 8$  mm,  $D = 50$  mm,  $E_{\text{ball}} = 210$  GPa,  $\nu_1 = 0.3$ ,  $E_{\text{race}} = 2.8$  GPa,  $\nu_{\text{race}} = 0.35$ , compressive strength  $p_{\text{m,max}} = p_c = 95$  MPa, top angle of the groove  $90^\circ$ . The effective contact radius  $R'$  can be approximated by the contact of a ball on a flat. This is a good approximation for large  $D/d$  ratio.

Calculate the effect of the ball diameter on the load capacity of the thrust bearing expressed in  $F(d_1)/F(d_2)$ .

#### Problem 4.5: Maximum contact pressure

Calculate the ratio between the load capacity of a steel ball and a ceramic ball running on a steel plane surface.

Steel ball: 100Cr6,  $E = 213$  GPa,  $\nu = 0.29$

Ceramic ball:  $\text{Si}_3\text{N}_4$ ,  $E = 300$  GPa,  $\nu = 0.28$

Steel plane surface: 100Cr6,  $E = 213$  GPa,  $\nu = 0.29$

Hint: The maximum contact pressure in the ceramic ball - steel contact is limited by the maximum contact pressure of steel, i.e.  $p_{\text{m,c}} \approx R_{\text{p0.2}}$



#### Problem 4.6: Calculation of the static safety factor to ensure smooth vibration free motion.

In ISO 76: 1987 the static load rating  $C_0$  of rolling bearings is defined by the maximum contact pressure in the most heavily loaded contact, for deep groove ball bearings  $p_{\text{max}} = 4.2$  GPa and for roller bearings 4 GPa (page 105). This contact pressure produces a permanent deformation of the rolling element and raceway, which is approximately  $1/10,000$  of the rolling element diameter. For smooth vibration free motion the required basic static load rating  $C_0$ , can be determined from  $C_0 \geq s_0 P_0$  where  $s_0$  is the static safety factor.  $P_0$  is the maximum load at which the contact deformation remains fully elastic. For ball bearing steel  $R_{\text{p0.2}} = 1.85$  GPa.

a) Calculate  $s_0$  for initial point contacts (eq. 4.9 page 105).

b) Calculate  $s_0$  for initial line contacts (eq. 4.26 page 110).

#### Problem 4.7: Contact stiffness

The contact radius  $r$  [m] of Hertzian point contact is related to the elastic indentation  $\delta$  [m] according (eq. 4.5 page 103). Calculate the factor to which the elastic indentation will increase when the load is doubled.

#### Problem 4.8: Contact stiffness

Many tooling machines are equipped with cast iron slide surfaces because of the superior bearing stiffness and shock resistance. Rolling guides have advantage over slide surfaces when low friction is required.

The axial stiffness of rolling guides (eq. 4.5 and 4.6 page 103) can be increased by preload. Calculate the stiffness ratio  $S'$  of a steel ball with diameter 5 mm running on a steel flat ( $E' = 230$  GPa), when loaded by

100 N preload and 10 N payload and when loaded by 10 N payload only. Payload is the load that is supported.

#### Problem 4.9: Fatigue life of cam-roller supports

The dynamic load rating of commercial cam-roller systems is based on a surface hardness of the raceways between 58 to 64 HRC. If the hardness is lower than this range, the basic dynamic load rating  $C$  needs to be multiplied by the respective hardness factor ( $f_H$ ). Calculate the factor of which the  $L_{10}$  fatigue life will decrease when the hardness of the support is HRC = 50 and  $f_H=0.6$ .

#### Problem 4.10: Material selection of gears

Calculate the ratio of the maximum driving torque that can be transmitted by a set of spur gears (line contact) made of 16MnCr5 with  $\sigma_{Hlim} = 1400$  MPa compared to a set made of 34CrMo4QT with  $\sigma_{Hlim} = 700$  MPa (Hint: consider eq. 4.26 page 110, eq 4.73 page 137 and Table 4.6 page 134).

#### Problem 4.11: Brinell hardness test

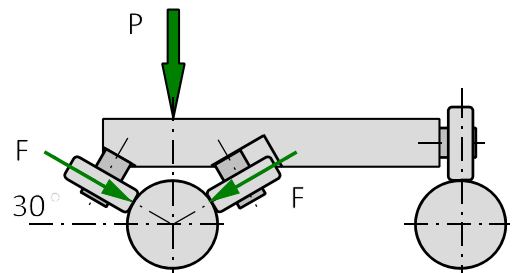
- In the Brinell Hardness test, the indentation made by a 10mm ball should lie between 0.24D and 0.6D. The load can be chosen in the range of 1, 2.5, 5, 10, 15 and 30kN. Examine the minimum and maximum hardness that can be measured for each load.
- Calculate the load  $F_c$  whereby deformation can still just be considered to be purely elastic. Do the calculation for a steel ball as well as a “hard metal” tungsten carbide ball. For the steel ball take  $E = 213$  GPa,  $\nu = 0.29$  and for the tungsten carbide ball  $E = 640$  GPa and  $\nu = 0.26$ . Assume that the material in the experiment has a hardness of 600HBW and that  $p_c = H/3$ . Explain the result of the calculation.
- Use the details in the previous question to calculate  $F/F_c$  for 600HBW,  $F = 30$  kN for both the steel ball and the hard metal ball. What can you conclude about the accuracy of the measurements?



#### Problem 4.12: Load carrying capacity of a roller guide

The load capacity of a guiding system needs to be calculated. The outer rings of the track rollers are made of ball bearing steel ( $R_{p0.2} = 1.85$  GPa,  $E = 206$  GPa,  $\nu = 0.3$ ). The rails are made of carbon steel ( $R_{p0.2} = 1080$  MPa, same  $E$  and  $\nu$ ). The diameter of the rollers and the rails are the same,  $D = 16$  mm.

- Calculate the load capacity of the guiding system when the deformation in the Hertzian contacts needs to remain fully elastic.
- During a running in phases a running track is created on the rail surface by plastic deformation. Calculate the load  $P$  that creates almost fully plastic deformation during running in ( $p_m = H$ ,  $H/R_{p0.2} = 3$ ).
- Calculate the track width (flattening of the rail) that remains after the running in phase.



### Problem 4.13: High speed hybrid ball bearing

Ceramic balls are applied in ball bearings to enable higher rotational speed. The lower density of the ceramic balls (silicon nitride) decline the centrifugal force.

- Find out by what percentage the centrifugal force  $F = -m\omega^2 R$  declines when using ceramic balls. By what percentage can the rotation speed be increased? The density of steel is  $7800 \text{ kg/m}^3$ , while the density of silicon nitride is  $3200 \text{ kg/m}^3$ .
- Because the E-modulus of silicon nitride is approximately 1.5 time that of steel, an equal load will lead to higher contact pressure. The advantage of a smaller centrifugal force is hence limited. How big is this effect? For ball bearing steel  $E = 213 \text{ GPa}$ ,  $\nu = 0.29$ , for silicon nitride  $E = 315 \text{ GPa}$ ,  $\nu = 0.26$ .
- By what percentage can the rotation speed be increased when both effects are combined?
- What other effects will be advantageous with respect to the higher rotation speed by using ceramic balls?

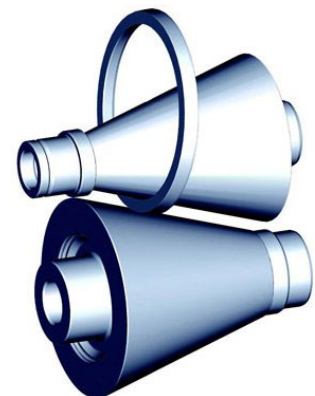


### Problem 4.14: Cone on ring CVT / Hertzian line contact subjected to rolling with traction

Consider the line contact of a cone on ring cvt subjected to rolling with traction, both components are made of carburised steel in accordance with DIN17210, with  $E = 200 \text{ GPa}$ ,  $\nu = 0.3$ ,  $R_{p0.2} = 835 \text{ MPa}$ . Consider the contra form contact between the ring and cone:  $R_1 = R_2 = 30 \text{ mm}$ ,  $L = 10 \text{ mm}$ . What is the maximum normal load

- for pure rolling under which no plastic deformation occurs?
- for rolling when the traction force amounts to 30% of the normal force and no plastic deformation is allowed to occur?

The normal load calculated above is based on the so called “static load rating”, it is the maximum load that can be transmitted without initiating plastic deformation. For surface durability reasons, the so called “dynamic load rating”, the fatigue strength and lubrication conditions need to be considered (Case 4.10 page 140).



### Problem 4.15: Surface durability of a gear set

The calculation of the surface durability is based on the Hertzian contact pressure  $p_{\max}$  which may not exceed the allowable stress number  $\sigma_{H,lim}$  of gear materials (Table 4.6 page 134).

- The contact pressure of a carburised and hardened spur gear set needs to be verified. The pinion is driven by a torque of  $T_1 = 750 \text{ Nm}$ . From the information on the drawing in accordance with NEN2366 follows that  $z_1 = 15$ ,  $z_2 = 71$ ,  $m = 7 \text{ mm}$ ,  $\alpha = 20^\circ$  and  $b = 44 \text{ mm}$ . Calculate  $p_{\max}$ .
- What would be the factor of safety for the driving torque when selecting a steel with  $\sigma_{H,lim}$  is 1.2 times the calculated value of  $p_{\max}$ .



### Problem 4.16: Contact friction

The friction force between two surfaces in sliding motion can be written as  $F_f = \tau A$  where  $\tau$  is the interfacial shear stress. The shear stress is assumed to be a constant and will be discussed in the next chapter. The contact area  $A$  of a Hertzian point contact is a function of  $F$  (eq. 4.10 page 105). Derive an equation for  $F_f(F) = C F^B$ . What is  $B$  and what is  $C$ ?

### Problem 4.17: True or Untrue?

Static and dynamic load rating of concentrated contacts

1. The Hertzian formulae are derived on the basis of 1) linear elastic deformation, 2) the stresses below the surface are not affected by finite dimensions of the contacting bodies and 3) the contacting surfaces are assumed to be frictionless (page 102).
2. Brinell hardness is measured as the depth of an indentation of a hard metal ball under prescribed load (page 106).
3. The maximum Hertzian contact load (elastic deformation) of a ceramic ball running on a steel flat is higher than that of a steel ball running on a steel flat, when the ball and flat are of the same steel grade (page 105, where  $F_c$  is proportional to  $p_{m,c}^3 / E'^2$ ).
4. The maximum Hertzian contact load of a point contact is proportional to the yield strength of the weakest body to the power three  $F \sim R_{p0.2}^3$  (page 105).
5. The Hertzian formulae can be applied for stationary concentrated contacts as well for rolling contacts.
6. In normally loaded concentrated contacts the contact pressure results in normal stresses which are the greatest on the surface (page 104).
7. First inelastic yielding in normally loaded concentrated contacts takes place just below the surface. This is where the shear stresses are maximum (page 104).
8. In normally loaded concentrated contacts the contact pressure at the surface leads to shear stresses which are the greatest at the surface (page 104).
9. The static load rating  $C_0$  of ball bearings, represents the maximum load at which the deformation in the Hertzian contacts remain fully elastic (page 105).
10. The static load rating of deep groove ball bearings is defined as the contact load at which a contact pressure occurs of 4.2 GPa. This contact pressure is the maximum contact pressure at which the deformation is still elastic (page 105).
11. The dynamic load rating of deep groove ball bearings is defined as the load at which a maximum contact pressure will occur of 4.2 GPa (page 115, 128).
12. In deep groove ball bearings the dynamic load rating  $C$  is smaller than the static load rating  $C_0$  (page 129).
13. Doubling the load on a deep groove ball bearing will decrease the fatigue life by a factor  $2^3=8$  (page 18).
14. Doubling the load on a roller bearing, which transmit the load via line contacts, will decrease the fatigue life by a factor  $2^{10/3}$  (page 18).
15. "Normal" loaded ball bearings are loaded in the range  $0.06C < P \leq 0.12C$ , where  $C$  is the dynamic load rating (page 19).

16. Ceramic balls applied in deep groove ball bearings running at high speed are favourable over steel balls because of the much lower centrifugal forces (page 129).
17. Rolling bearings in motion, loaded up to the static load rating show initial plastic deformation which is limited to the running in period (page 102).
18. The static load rating of deep groove ball bearings could be increased significantly when bearing manufacturers would increase the contact conformity between the balls and the raceway, by adapting the groove radius of the raceway.

#### Rolling friction, rolling with traction, stress concentrations, premature failure

19. The slip between the side surfaces of a cylinder roller with its track is typically an example of Heathcote slip (page 116).
20. Reynolds slip typically occurs in a rolling contact under traction (page 116).
21. In the ball-raceway contact of angular contact ball bearings a combination of rolling and spinning takes place (page 119).
22. Sub-surface initiated cracks normally progress transversely to the surface (page 113).
23. Surface initiated fatigue cracks normally progress parallel to the surface (page 113).
24. Properly designed and correctly mounted ball bearings will finally fail by sub-surface initiated fatigue in general (113).
25. The surface durability of gears is limited by surface initiated fatigue (113).
26. The rolling resistance in a steel ball – ring contact is predominantly determined by the time dependent elastic recovery, called hysteresis loss (page 118).
27. Plastic deformation in concentrated contacts of rolling mechanisms may vanish during running in by work hardening of the steel and an increase of contact conformity by inelastic yielding (page 102).
28. The surface durability (fatigue strength) in gear transmissions is based on the Hertzian formulae for concentrated contacts (page 133).

#### Lubrication

29. In rolling bearings the balls and raceways become separated during service by elasto-hydrodynamic lubrication, dependent on load, speed and surface roughness. It means that there is no mechanical contact between the balls and the rings during surface (page 122).
30. Relative high loaded ball bearings running at low velocity are boundary- or mixed lubricated (page 122).
31. Traction drives in which the traction force should be transferred via concentrated contacts will fail by macro slip, when elasto hydrodynamic lubrication (EHL) exists (page 138).



## Solutions

## 4.1)

a) The effective contact radius is calculated using eq. 4.3 page 103

$$i) \quad r_{1x} = \infty, r_{1y} = r_1, r_{2x} = r_1, r_{2y} = \infty: R' = r_1/2 = 2.5 \text{ mm}$$

$$ii) \quad r_{1x} = \infty, r_{1y} = r_1, r_{2x} = r_1, r_{2y} = -r_2: R' = 4.583 \text{ mm}$$

The minus sign is explained by the concave surface as explained on page 103.

b) If the cam roller diameter is equal to the diameter of the rail guide, then the radius of the Hertzian contact  $r_x = r_y = r$ . Substitution of  $p_m = p_{\max}/1.5$ ,  $p_{\max} = 4.2 \text{ GPa}$ ,  $R' = 2.5 \text{ mm}$  and  $E' = 233 \text{ GPa}$  in eq. 4.10 page 105 results in the static load rating of the point contact of  $C_0 = 708 \text{ N}$ . The contact pressure of  $4.2 \text{ GPa}$  corresponds to a plastic indentation of  $1/10,000$  of the roller diameter.

If the rolling contact should remain fully elastic then the load capacity is limited by  $p_{m,c} = R_{p0.2} = 1850 \text{ MPa}$  and results in  $F_c = 204 \text{ N}$ , a factor 3.5 less. This can be written as  $C_0 = s_0 F_c$  where  $s_0 = 3.5$ .

## 4.2)

a) For the point contact  $r_{1x} = R_p, r_{1y} = R_p, r_{2x} = -R_b, r_{2y} = -R_b: R' = 1/(2/R_p - 2/R_b)$

b) For the point contact  $r_{1x} = R_p, r_{1y} = \infty, r_{2x} = -R_b, r_{2y} = \infty: R' = 1/(1/R_p - 1/R_b)$

## 4.3)

Inner diameter raceway:  $r_{1x} = \frac{d}{2}, r_{1y} = r_{1x}, R_{\text{race}} = \frac{D\sqrt{2}}{2}, r_{2x} = \infty, r_{2y} = R_{\text{race}}$  in (eq.4.3) gives  $R' = 0.972 \text{ mm}$

Outer diameter raceway:  $r_{1x} = \frac{d}{2}, r_{1y} = r_{1x}, R_{\text{race}} = \frac{D\sqrt{2}}{2}, r_{2x} = \infty, r_{2y} = -R_{\text{race}}$  in (eq.4.3) gives  $R' = 1.029 \text{ mm}$

## 4.4)

$$n_1 = \text{int}\left(\frac{\pi D}{d_1}\right) = 39 \quad n_2 = \text{int}\left(\frac{\pi D}{d_2}\right) = 19$$

$$p_m = \frac{F}{\pi r^2} \text{ substitution of } r \text{ from (eq.4.3) gives } \left(\frac{F_1}{F_2}\right) = \left(\frac{R'_1}{R'_2}\right)^2$$

$$\left(\frac{R'_1}{R'_2}\right)^2 = \left(\frac{d_1}{d_2}\right)^2 \quad \frac{F_{1b}}{F_{2b}} = \frac{n_1 d_1^2}{n_2 d_2^2} = 0.513$$

4.5) The load capacity is calculated using eq. 4.10 page 105. The critical value of the maximum contact pressure is limited by the steel surface for both material combinations. The parameter that matters in this equation is the effective E-modulus (eq. 4.2 page 103):

$$\left. \begin{array}{l} E'_{\text{steel-steel}} = 233 \text{ GPa} \\ E'_{\text{steel-ceramic}} = 271 \text{ GPa} \end{array} \right\} \frac{F_{\text{steel-steel}}}{F_{\text{steel-ceramic}}} = \left( \frac{E'_{\text{steel-ceramic}}}{E'_{\text{steel-steel}}} \right)^2 = 1.36$$

## 4.6)

a) For point contacts and elliptic contacts:  $C_0$  is based on the maximum contact pressure  $p_{\max} = 4.2 \text{ GPa}$  (according ISO 76: 1987 page 105). The mean contact pressure is  $p_m = p_{\max}/1.5$ . First inelastic yielding takes place when  $p_m = R_{p0.2}$ . For ball bearing steel  $R_{p0.2} = 1.85 \text{ GPa}$ . The maximum contact load is proportional to  $p_{m,c}^3$  (eq. 4.10 page 105).

$$s_0 = \frac{C_0}{P_0} = \left( \frac{p_{\max}/1.5}{R_{p0.2}} \right)^3 = 3.5$$

When running in the same track the plastic deformation will vanish in a few load cycles.

- b) For line contacts:  $C_0$  is based on  $p_{\max} = 4$  GPa (according ISO 76: 1987 page 105). The mean contact pressure  $p_m = (\pi/4)p_{\max}$ . First inelastic yielding takes place when  $p_m = (0.5/0.387)R_{p0.2}$ . The maximum contact load is proportional to  $p_m^2$  (eq. 4.26 page 110).

$$s_0 = \frac{C_0}{P_0} = \left( \frac{\frac{\pi}{4} p_{\max}}{\frac{0.5}{0.387} R_{p0.2}} \right)^2 = 1.7$$

Rolling elements with line contacts and loaded up to  $C_0$  will fail by ratcheting (page 115).

$$4.7) \quad r = \left( \frac{3FR'}{E'} \right)^{1/3} = (2R'\delta)^{1/2} \quad \delta = CF^{2/3} \quad 2^{2/3} = 1.587$$

- 4.8) The contact stiffness is related to indentation (eq. 4.6 page 103), with the contact radius (eq. 4.5 page 103) and with the load (eq. 4.1 page 103):

$$\frac{S_1}{S_2} = \left( \frac{\delta_1}{\delta_2} \right)^{0.5} \quad \frac{\delta_1}{\delta_2} = \left( \frac{r_1}{r_2} \right)^2 \quad \frac{r_1}{r_2} = \left( \frac{F_1}{F_2} \right)^{1/3}$$

This can be written as  $S(F)$ :

$$\frac{S_1}{S_2} = \left( \frac{F_1}{F_2} \right)^{1/3} = \left( \frac{110}{10} \right)^{1/3} = 2.224$$

The contact stiffness has increased by a factor 2.224. The fatigue life however will decrease dramatically with this heavy preload, starting and running friction will also be higher and as a consequence the bearing temperature will be higher. To eliminate running noise in ball bearings an axial preload is advised of 0.01C (a mean contact pressure of 1 GPa).

- 4.9) With a surface hardness of HRC50 the dynamic load rating will decrease from C to 0.6C. The fatigue life will decrease with  $(1-0.6^3) 100\% = 78.4\%$  (eq. 1.14 page 18).

- 4.10) From eq. 4.73 page 137 it follows that the maximum contact load is related to  $\sigma_{Hlim}^2$ . This results for the maximum driving torque:

$$\frac{T_1}{T_2} = \frac{F_{t1}}{F_{t2}} = \left( \frac{\sigma_{Hlim1}}{\sigma_{Hlim2}} \right)^2 = \left( \frac{1400}{700} \right)^2 = 4$$

Note that the Hertz equation for line contacts (eq. 4.26 page 110) would give the same result.

#### 4.11)

- a) Substitution of the numerical data in eq. 4.14 page 106 gives HBW in  $10^7$  MPa ( $30HB \approx 300MPa$ )

	1	2.5	5	10	15	30 kN
d=0.6D	3	8	16	32	48	95HBW
d=0.24D	22	54	109	218	327	653HBW

- b) Substitution of the numerical data in (eq. 4.10 page 105) gives:

steel - steel	$E' = 233GPa$	$F_c = 258N$	$F/F_c = 116$
steel - diamond	$E' = 347GPa$	$F_c = 116N$	$F/F_c = 260$

The hard metal ball flattens less and consequently has a smaller contact surface as a function of the load.

- c) With a steel ball it appears that  $F/F_c < 200$ , indicating that the deformation is not fully plastic (Figure 4.4 page 105). Because of the flattening of the steel balls these are unsuitable to determine the hardness of hard objects ( $>450\text{HBW}$ ).

#### 4.12)

- a) The Hertzian load is limited by the yield strength of the rail guides. The Hertzian contact will be a circular point contact with radius  $r$  since the roller diameter and the diameter of the rail guide are the same. Substitution of  $p_{m,c} = R_{p0.2} = 1080\text{ MPa}$  and  $R' = 4\text{ mm}$  in (eq. 4.10 page 105) results in  $F_c = 110\text{ N}$ . The maximum value of the payload becomes  $P = 2 F_c \sin 30 = F_c$ .
- b) It is shown in Figure 4.4 page 105 that the relation  $p_m \sim F^{1/3}$  according Hertz holds reasonably good up to  $p_m = H$ . Substitution of  $p_m = H$  in (eq. 4.10 page 105) results in  $F_H = 3^3 F_c$ .
- c) The contact area  $A = F_H / H = 0.915\text{ mm}^2$ . With by approximation a circular contact the contact width  $2r = 1.08\text{ mm}$ .

#### 4.13)

- a) The weight of a silicon nitride ball is 60% less than that of a steel ball, resulting in a centrifugal force that is 60% lower. The rotation speed could consequently be increased by a factor of  $\omega_2/\omega_1 = (1/0.6)^{1/2} = 1.3$  or 30%.
- b) For steel-steel  $E'_1 = 230\text{ GPa}$ , for steel-silicon nitride it follows that  $E'_2 = 272\text{ GPa}$ . The maximum contact pressure is constrained by the less hard steel ring. For the load ratio of both material combinations, it follows from (eq. 4.19 page 109) that:  $F_{\text{ceramic}}/F_{\text{steel}} = (E'_1/E'_2)^2 = 0.7$ . This means the maximum load for the steel-silicon nitride contact is 30% less.
- c) A 60% lower centrifugal force and a 30% lower load rating therefore means the advantage of the lower density will be halved. The rotational speed can consequently be increased not by 30% for ceramic balls but only by 15%.
- d) Ceramic balls create a smaller contact surface and hence a lower resistance to rolling with less heat development as a result. This positively affect the maximum rotational speed for hybrid ball bearings, which would in total lie between 15% and 30% higher than for steel ball bearings.

#### 4.14)

- a) The maximum contact load of a line contact in pure rolling is given by eq. 4.24 page 110. Substitution of  $p_m = p_{m,c} = (0.5/0.387)R_{p0.2}$ ,  $R' = 15\text{ mm}$  and  $E' = 220\text{ GPa}$  gives  $F_c = 8.1\text{ kN}$ .
- b) For rolling with traction and  $F_{\text{tan}}/F \Rightarrow 1/9$  the maximum shear stress occurs at the surface. From Table 4.2 page 114 it follows with  $F_{\text{tan}}/F = 0.3$  that  $\tau_{\text{max}}/p_m = 0.51$ . Substitution in eq 4.25 page 110 gives  $p_{m,c} = 0.5/0.51 R_{p0.2}$  and from eq. 4.10 page 109 that  $F_c = 4.7\text{ kN}$ . This means the maximum normal force has decreased by 42%. The drive torque that can be applied is  $M = 0.3 \cdot F_c \cdot R = 41.9\text{ Nm}$ .

#### 4.15)

- a) Consider eq. 4.68 page 135. Firstly, factors  $Z_H$ ,  $Z_E$  and  $Z_\epsilon$  are determined. From (eq. 4.70 page 136) it follows that  $Z_E = 1.9 \cdot 10^5\text{ Pa}^{1/2}$  and  $Z_H = 2.495$ . To establish  $Z_\epsilon$  (4.79), the necessary gear dimensions must be determined first (eq. 4.66 page 135). For the contact ratio, it follows that  $\epsilon = 1.646$  and for the contact ratio factor  $Z_\epsilon = 0.886$ . The tangential force follows from  $T = F_t d_1/2$ , this is  $F_t = 14.3\text{ kN}$ . Substitution of the numerical data in (eq. 4.68 page 135) gives  $p_{\text{max}} = 0.8\text{ GPa}$ .
- From Table 4.6 page 134 it follows for carburised and hardened steel that  $p_{\text{max}} = 1.3 \dots 1.5\text{ GPa}$ . It means that the fatigue strength of the tooth flanks may be sufficient. The impact of imperfect alignment, peak loads from the drive and other possible effects still have to be incorporated into the calculation.

- b) The relation between the limiting contact stress  $\sigma_{Hlim}$  and the maximum tangential force is presented in (eq. 4.73 page 137), i.e.  $\sigma_{Hlim} \sim F_t^{1/2}$ . Because  $T_{max} \sim F_t$ , it follows  $\sigma_{Hlim} \sim T_{max}^{1/2}$ . With  $\sigma_{Hlim} = 1.2 \cdot p_{max}$  a factor of safety for the driving torque would be  $1.2^2 = 1.44$ .

4.16) From eq. 4.10 page 105 it follows

$$F_f = \tau A \quad A = \frac{F}{p_m} \quad p_m = C_1 F^{1/3} \quad C_1 = \frac{1}{\pi} \left( \frac{1}{3} \right)^{2/3} \left( \frac{E'}{R'} \right)^{2/3}$$

$$F_f = \tau \frac{F}{C_1 F^{1/3}} = \frac{\tau}{C_1} F^{2/3} = C_2 F^{2/3} \quad C_2 = \frac{\tau}{C_1} \quad \mu = \frac{C_2}{F^{1/3}}$$

The contact area A and  $F_f = \tau A$  increases less than proportional with F.

The coefficient of friction is a constant when  $F_f$  increases proportional with F.

For the spherical contact the coefficient of friction decreases with F.

4.17) True / Untrue

True 1, 4, 5, 6, 7, 13, 14, 15, 16, 17, 18, 19, 21, 24, 25, 27, 28, 29, 30

Untrue 2, 3, 5, 8, 9, 10, 11, 12, 20, 22, 23, 26, 31

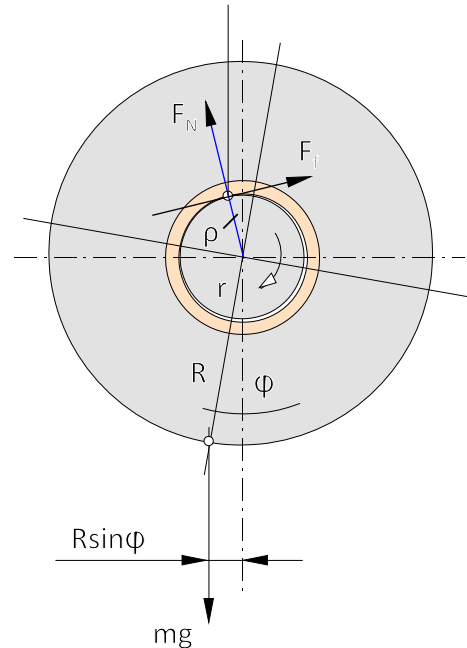
## Chapter 5 - Problems with Solutions

## Problem 5.1: Measurement of bearing friction

The friction in a sliding bearing is measured using a ring element hanging over a shaft. The bearing is mounted in the bore of the ring. In point A at the outer surface of the ring the bearing load is applied using a mass hanging on the flexible cord.

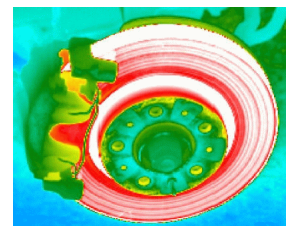
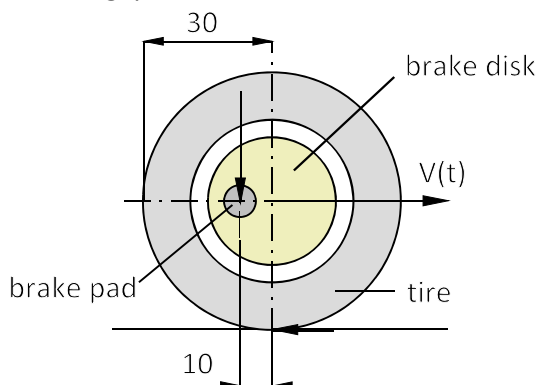
If the shaft is driven clockwise then point A will take a position left from the centerline as illustrated.

- Derive an equation to calculate the coefficient of friction as a function of  $\varphi$ , the ring diameter  $D$  and the shaft diameter  $d$ .
- Calculate the coefficient of friction when  $\varphi=3.75$  deg and  $R=3r$ .



## Problem 5.2: Frictional heating of a disk brake

In order to save weight, pearlitic cast iron disk brakes could be replaced by aluminium disks. Aluminium is also a better heat conductor than cast iron. With surface treatment, the aluminium can be made very resistant to wear. A possible disadvantage is that the maximum allowable temperature of aluminium is lower than that of cast iron. Examine the performance of an aluminium brake disk by answering the following questions.



- If the car comes to a stop from a driving speed of  $V_t=100$  km/h with a constant deceleration, the braking distance appears to be  $S = 100$  m (standard for cars). What is the deceleration before the stop and how long will it take to stop?
- The mass of the car is  $m = 1000$  kg. What is the required friction coefficient  $\mu$  between the tires and the road surface to prevent slip?
- Approximately 80% of the braking energy is absorbed by the front wheels. Both front wheels are equipped with a single brake disk and a brake pad on each side of the disk. The wheel diameter is 0.6 m. The distance between the centre of the brake pad and the wheel centre is 0.1 m. The friction coefficient between the brake disk and the brake pad is  $\mu = 0.4$ . How great is the pressure force on the brake pads?

- d) What temperature does the brake disk reach when all friction energy is absorbed by the brake disk? Consider both the cast iron and aluminium brake disks. The brake disk has a diameter of 300 mm and a thickness of 12 mm. For cast iron  $\rho = 7300 \text{ kg/m}^3$ ,  $c = 0.50 \text{ kJ/(kg K)}$ ,  $k = 60 \text{ W/(m K)}$ ,  $T_{\text{melt}} = 1450 \text{ K}$ , while for aluminium  $\rho = 2700 \text{ kg/m}^3$ ,  $c = 0.88 \text{ kJ/(kg K)}$ ,  $k = 237 \text{ W/(m K)}$ ,  $T_{\text{melt}} = 932 \text{ K}$ .

Note: Since braking only occurs for short periods, little heat will be dissipated through convection. The brake disk absorbs much more heat than the pads, firstly because the contact surface of the brake disk is much bigger (non-stationary contact) and also because it is a better conductor for heat than the pads. For a first approximation, it is justifiable to assume that all heat goes into the disk.

---

### Problem 5.3: Tire width Formula 1 - racing car

The first classical law of friction states: “The size of the contact surface does not influence the friction”. Why, then, are tires for the Formula 1 racing cars so wide?

According to the “FIA Technical Commission”, the organisation that decides on the technical rules for Formula 1, the front tires should not be wider than 355 mm (i.e. 14 inches) and the rear tires no wider than 380 mm (15 inch).

---

### Problem 5.4: Acceleration of Formula 1 racing car <sup>1</sup>

A modern Formula 1 racing car manages approximately 1.5 km/liter (or 1 :1.5). To complete the Grand Prix distance, it would need approximately 180 liters of fuel. Since the time gain from a lower mass exceeds the disadvantage of a refuelling stop, the tank contents are much smaller. The weight of a car, including a full tank and the driver, is approximately 600 kg. A Formula 1 car accelerates six to ten times as quickly as a normal car. Acceleration of 0-100 km/h in 2 seconds, 0-160 km/h in 3.5 seconds and 0-250 km/h in less than 6 seconds is typical. The top speed of an F1 racer is around 370 km/h. Accelerating from 0-160 km/h and braking to a complete halt takes approximately 6.5 seconds. This means the car speeds up and slows down at the same rate. What is the required friction coefficient between the tire and the road surface to make this acceleration possible?

---

### Problem 5.5: True or untrue?

Coulomb's friction law and its exceptions

1. Coulomb's friction law - if the load is doubled the friction is doubled - can be explained by the elastic deformation in the micro contacts, i.e. in the real contact area (page 150).
2. In polymer-metal contacts the real contact area increases more than proportional with the load which explains the larger friction coefficient at higher load (page 150).
3. Coulomb's friction law, that says that the friction in a dry sliding contact is independent of the size of the visible area, do not apply to polymers (page 150).
4. The real contact area is formed by the micro contacts between roughness peaks of the surfaces. Friction in dry sliding or boundary lubricated contacts is exclusively a result of ploughing and adhesion forces in these micro contacts (page 150).

---

<sup>1</sup> Test Method for Friction Coefficient Measurements between tire and pavement using a variable slip technique ASTM E 1859-97 and E 1844-96

5. With a Brinell hardness of HB300, the maximum contact pressure is 3000 MPa (300 kgf/mm<sup>2</sup>) (page 107).
6. The friction in dry sliding polymer - metal contacts is independent of the sliding velocity (page 158).
7. The friction loss in a polymer plain bearings loaded with a normal PV-value is larger when using a stainless steel shaft, compared to a carbon steel shaft (page 158).
8. The coefficient of friction in polymer metal contacts decreases with the contact pressure (page 150).
9. The friction force in polymer metal contacts decreases with the contact pressure (page 150).
10. Friction between hydrodynamically lubricated smooth surfaces increases with the surface velocity (page 158).

---

#### Problem 5.6: True or untrue?

How to manage friction by ploughing

1. The friction force by ploughing between materials of different hardness depends on the radius of the asperities (roughness peaks) of the harder surface. A large radius is optimal (page 151).
2. The friction force by ploughing can be decreased by super finishing techniques in which the radius of the asperities is enlarged (making the asperities blunt) (page 151).
3. Super finishing techniques only smoothens the roughness of the preceding cutting process (page 147).
4. The friction by ploughing is minimal for infinite smooth surfaces (page 158).
5. The ploughing component of friction in steel-steel contacts can be decreased by selecting materials of the same high hardness (page 151).
6. When two bodies of different hardness are in contact, the friction by ploughing is dominated by the roughness of the less harder surface (page 152).
7. In polymer - metal contacts the total friction remains minimal when the polymer surface has a roughness of around Ra=0.2 µm (page 156, 158).



---

**Problem 5.7: True or untrue?**

How to manage friction by adhesion

1. Metallurgical compatible (-soluble) materials are to be selected to minimize the tendency to adhesive wear (page 154).
2. Alloyed steel is generally beneficial in preventing adhesive wear, because it reduces surface energy and at the same time increases hardness (small  $\gamma/H$ ) (page 156).
3. One of the measures to eliminate stick-slip in polymer - steel contacts is by making the steel surfaces smoother (super finishing) (page 152, 159).
4. The coefficient of friction is always smaller than  $\mu \leq 1$ , the asymptotic value of the coefficient of friction (page 159).
5. The friction in zinc plated fasteners is limited by the low shear strength of the zinc (page 155)
6. The adhesion component of friction can be decreased by applying a thin soft coating (small  $\tau/H$ ) (page 155).
7. The friction between two extremely smooth surfaces is dominated by molecular attraction between the atoms of both surfaces (page 152).
8. The predominant cause that two surfaces may stick is adhesion (page 152).
9. Stainless steel fasteners are sensitive to adhesive friction by the very thin oxide layer on stainless steel that is easily rubbed off (oxide layer thickness  $< 10$  nm) (page 155).
10. The coefficient of friction between metals operating in vacuum environment is much higher than in the regular atmosphere, which can be explained by the protective oxide layer that cannot be regenerated in vacuum (page 155).
11. Polymer - steel contacts do not need to be lubricated, but lubrication can help to reduce friction and wear.
12. The combination of two ceramics in dry sliding show LARGER adhesive friction than steel - ceramic combinations.

---

**Problem 5.8: True or untrue?**

Surface characterization

1. The  $R_a$  surface roughness sometimes called the Arithmetical mean or Centre Line Average is independent of the length scale of the surface profile. For example, a surface profile described by  $\sin(x)$  and  $\sin(2x)$  respectively would result in the same  $R_a$ -value (page 145).
2. None of the roughness values  $R_a$ ,  $R_t$ , RMS are dependent on roughness spacing. If the surface is stretched the same roughness value remains (page 145).

## Solutions

5.1) Sum of the moments:  $F_f r = mg R \sin(\varphi)$   $F_f = mg \frac{R}{r} \sin(\varphi)$

Normal load in the contact point:  $F_N = mg \cos(\rho)$

Coefficient of friction:  $\mu = \frac{F_f}{F_N} = \frac{R \sin(\varphi)}{r \cos(\rho)}$  with  $\mu = \tan(\rho)$  gives  $\sin(\rho) = \frac{R}{r} \sin(\varphi)$

Rearranging:  $\mu = \tan(\arcsin(\frac{R}{r} \sin(\varphi))) = \tan(\arcsin(\frac{D}{d} \sin(\varphi)))$

Approximate solution:  $mg R \sin(\varphi) = \mu mgr$   $\mu = \frac{R \sin(\varphi)}{r} = \frac{D \sin(\varphi)}{d}$

- b) With  $\varphi = 3.75$  deg then the exact solution gives  $\mu = 0.2$  and the approximate solution gives  $\mu = 0.196$ , it is 1.95 % underestimated.

In this measurement method the effect of the load on the coefficient of friction can be illustrated clearly, since  $\varphi$  is related to the coefficient of friction only.

## 5.2)

a)  $a = -\frac{1}{2} \frac{V_t^2 - V_0^2}{s}$ ,  $t = \frac{V_t - V_0}{a}$ ;  $a = -3.858 \text{ m/s}^2$ ,  $t = 7.2 \text{ sec}$

b)  $F_a = ma = 3858 \text{ N}$   $F = mg = 9.81 \text{ kN}$   $\mu = F_a / F = 0.393$

c)  $F_{\text{wheel}} = \frac{0.8 F_a}{2}$ ,  $\mu F_{\text{pad}} = \frac{30}{10} \frac{F_{\text{wheel}}}{2}$ ,  $\mu = 0.4$ ;  $F_{\text{pad}} = 5787 \text{ N}$

d)  $E = \frac{1}{2} m V_t^2 = 386 \text{ kJ}$

cast iron:  $m_{\text{schijf}} = \rho V = 6.2 \text{ kg}$   $Q = m_{\text{schijf}} c dT = E$   $dT = 125 \text{ K}$

aluminium:  $m_{\text{schijf}} = \rho V = 2.3 \text{ kg}$   $Q = m_{\text{schijf}} c dT = E$   $dT = 191 \text{ K}$

The aluminium brake disk proves to be warmer after braking once than the cast iron brake disk. Furthermore the maximum allowable temperature of the aluminium is much lower. It means that the heat conduction of the brake disk must be fully exploited for it to make aluminium a suitable material for the purpose.

From further research with thermal network analysis and practical experiments it appears that wheel bearings get substantially warmer due to the good heat conductivity of aluminium, which makes them critical components. The carbon fibre brake disks used in Formula 1 can be loaded up to 1000 degrees Celsius.

- 5.3) The classical friction laws apply to materials that undergo mainly plastic deformation in the asperity summits. Polymers display a lower friction coefficient at a high nominal contact pressure. A high friction force can consequently only be achieved with a large surface area. Other advantages of a wide surface are that the wear is distributed over a larger area, heat development per unit of surface is lower and the result is a larger heat-radiating surface.

In machine construction, where low friction is mostly considered desirable, high surface pressure are desirable. For instance, halving the diameter of a shaft can double the surface pressure. If the friction force was in proportion to  $p^{2/3}$ , the friction coefficient would be reduced to 63% of the original value. The friction torque would consequently be reduced to 31.5% of its original value!

- 5.4) To arrive at  $V=100$  km/h in  $t=2$  seconds, an acceleration is needed of  $a=V/t=14\text{m/s}^2$ . To this end a traction force of  $F_f=ma$  is needed. The necessary traction coefficient follows from  $\mu=F_f/F=ma/mg \approx 1.4$ . The distance required to reach the speed is then only  $S=\frac{1}{2}at^2=28\text{m}$ .

When accelerating and braking, the coureur experience an acceleration of approximately  $1.4g$ . When turning a corner at speed, drivers can only undergo up to  $4g$  for a few seconds before they become unconscious. Accelerations are limited by a prescribed maximum tire width with four circumferential grooves. Wet weather tires also have axial grooves to dissipate water sideways.

5.5) Coulomb's friction law and it's exceptions

1. Coulomb's friction law - if the load is doubled the friction is doubled - can be explained by the ~~elastic~~ plastic deformation in the micro contacts, i.e. in the real contact area (U).
2. In polymer-metal contacts the real contact area increases ~~more~~ less than proportional with the load which explains the ~~larger~~ smaller friction coefficient at higher load (U).
3. Coulombs friction law, that says that the friction in a dry sliding contact is independent of the size of the visible area, do not apply to polymers (T).
4. The real contact area is formed by the micro contacts between roughness peaks of the surfaces. Friction in dry sliding or boundary lubricated contacts is exclusively a result of ploughing and adhesion forces in these micro contacts (T).
5. If the Brinell hardness is HB300, then the maximum contact pressure is 3000 MPa ( $300 \text{ kgf/mm}^2$ )(T).
6. The friction in dry sliding polymer - metal contacts ~~is independent~~ increases with the sliding velocity (U)
7. The coefficient of friction in polymer metal contacts decreases with the contact pressure (T).
8. The friction force in polymer metal contacts ~~decreases~~ increases with the contact pressure (U).
9. Friction between hydrodynamically lubricated smooth surfaces increases with the surface velocity (T).

5.6) Ploughing True 1, 2, 3, 4, 5, Untrue 6, 7

5.7) Adhesion True 2, 5, 6, 7, 8, 9, 10, 11, Untrue 1, 3, 4, 12

5.8) Surface characterization True 1, 2

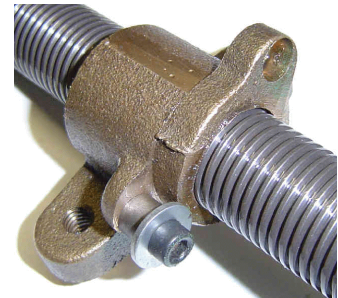
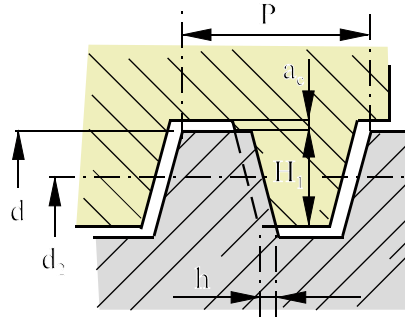
## Chapter 6 - Problems with Solutions

## Problem 6.1: Service life of a lead screw

Consider a lead screw assembly that operates with a contact pressure in the threaded area of  $p = 5 \text{ MPa}$ . The pitch diameter  $d_2 = 10.5 \text{ mm}$ . The stroke over which the nut is displaced is 20 times the nut height. The number of loaded turns during service life is  $n = 100 \cdot 10^3 \text{ rev}$ .

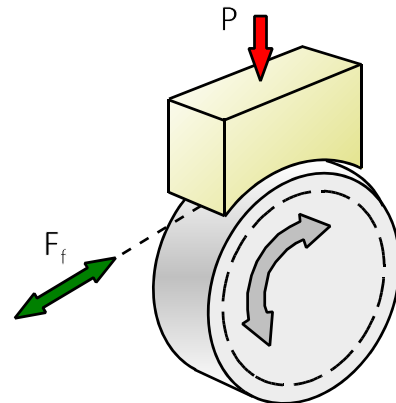
The specific wear rate of the bronze nut and the spindle are  $k_{\text{nut}} = k_{\text{spindle}} = 10 \cdot 10^{-15} \text{ m}^2/\text{N}$ . Calculate the increase of backlash  $h$  [mm] in the screw-nut interface of a trapezoidal lead screw

- caused by wear of the bronze nut.
- caused by wear of the spindle.



## Problem 6.2: Investigation to hard wearing materials for knee replacements

In order to assess the wear performance of different materials of total knee replacements (TKR), a block on ring test rig will be used. The ring is actuated in reciprocating motion.



- Calculate the required test duration in hours.

The ring is made of steel, the block from ultra high weight molecular polyethylene (UHMWPE). The density of UHMWPE is  $\rho = 945 \text{ kg/m}^3$ . A specific wear rate  $k = 10 \cdot 10^{-15} \text{ m}^2/\text{N}$  of the PE block is expected. A minimum wear of the polymer block of 0.1 gram is to be obtained to establish the wear rate. The contact surface  $A = 100 \text{ mm}^2$ , the surface pressure is  $p = 2 \text{ MPa}$ , the total sliding distance in one cycle is  $s_1 = 30 \text{ mm}$  and  $n = 2$  cycles per second are made.

- What temperature will the ring get when the frictional heating is to be transferred by convection only.

The coefficient of friction  $\mu = 0.12$ , the heat convection coefficient of the rotating disc in free air  $h_c = 80 \text{ W/m}^2\text{K}$  and the effective heat convection surface area of the ring  $A = 7 \cdot 10^{-3} \text{ m}^2$ .

### Problem 6.3: Service life of a linear axis using plain bearings

A linear guide's travel smoothness and tolerance variations are key concerns for machine designers. But, the most important design factor is how well the guide resists deflection. Linear support rails in combination with open design bearings are best suited to sustain heavy loads and to provide high stiffness.

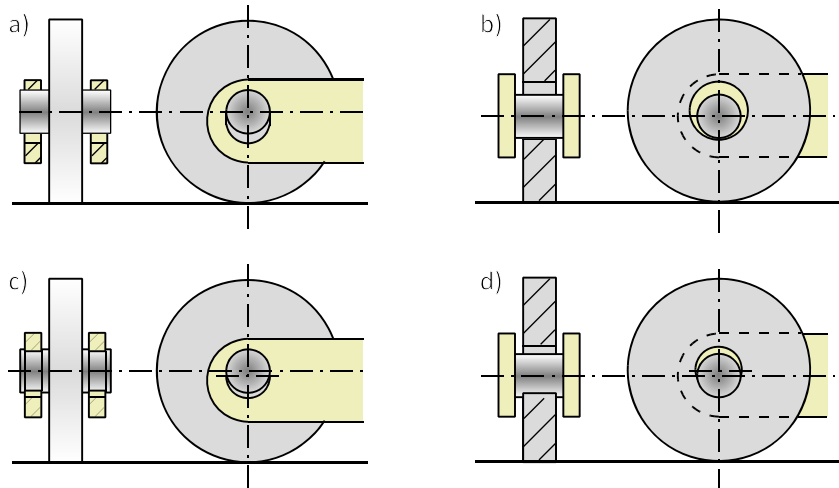
Linear plain bearings are the better choice compared to linear ball bearings when the bearing arrangement is subjected to heavy shock loads, vibrations or high accelerations in the unloaded state however, increased friction must be expected.



Calculate over what sliding distance  $s$  [km] the bearing will wear down over  $h=0.1$  mm. Consider a mean value of the contact pressure of  $p = 3$  MPa and the specific wear rate of  $k = 10^{-15}$  m<sup>2</sup>/N. Consider good conformity between the plain bearing and the linear support.

### Problem 6.4: Service life plain bearings

Consider the configurations with plain bearings and gravity loading as illustrated. The diameter of the shaft  $d$  and the total bearing length is  $L$  in all configurations. Calculate the wear volume when a shaft displacement of  $\delta$  is allowed.



- In configuration a) the wear of the shaft is minimal, the wear of the bearing predominates.
- In configuration b) the wear of the shaft is minimal, the wear of the bearing predominates.
- In configuration c) the shaft is predominantly subjected to wear, the wear of the bearing is minimal.
- In configuration d) the shaft is predominantly subjected to wear, the wear of the bearing is minimal.
- If the wear rate of the bearing in config a) is  $k$  and in config b) it is  $5k$ . Which of these bearings will last longer, the bearings in config a) or in config b)?
- Give a reasonable explanation for relative large  $k$  that might be expected in config b) with respect to config a).

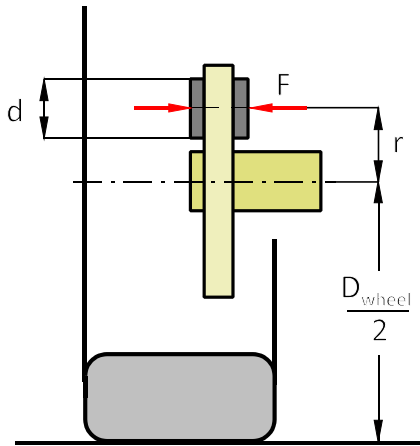
### Problem 6.5: Service life plain bearings

Consider a cooling fan with vertical shaft position and equipped with slide bearings. The rotor is not perfectly balanced which results in a rotating radial bearing load of  $F = 1 \text{ N}$ . The shaft diameter is  $d = 3 \text{ mm}$ . The bearing length  $L = d$ . The specific wear rate  $k = 10^{-15} \text{ N/m}^2$ .



- Calculate the increase of radial bearing play by wear after one year operation at 1400 rpm (365 days, 24 hours a day).
- Calculate the power loss in the bearing. Consider a coefficient of friction of  $\mu = 0.1$

### Problem 6.6: Service life disk brake

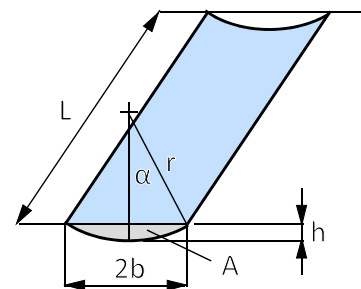


- Calculate the service life of disk brake pads, expressed in numbers of brake times. The contact area of the brake pad is approached by a round disk with a diameter of  $d=60\text{mm}$  located at centre distance  $r=100\text{mm}$ , the thickness of the brake lining is  $t=10\text{mm}$ , the specific wear rate  $k=50 \cdot 10^{-15} \text{ m}^2/\text{N}$  (class 5), the normal force  $F=6000\text{N}$ , the wheel diameter  $D=0.6\text{m}$  and the brake distance  $S=100 \text{ m}$ .
- How much thinner will the brake disk worn down during the service life of the brake pads if the specific wear rate of the brake disk equals that of the brake lining? Hint: What is  $h_{\text{disk}}$  if  $h_{\text{pad}} = t = 10 \text{ mm}$ .
- What is the perfect ratio of  $k_{\text{pad}}/k_{\text{disk}}$  that makes that the pad and disk are worn after the same sliding distance if  $h_{\text{pad}}/h_{\text{disk}} = 5$ ?

### Problem 6.7: Service life plain bearings

Consider the nominal line contact formed by a beam in contact with a rotating shaft. A scar width of  $2b = 4 \text{ mm}$  is created after 5000 revolutions of the shaft, the diameter of the shaft is  $20 \text{ mm}$ , the contact length  $L = 8 \text{ mm}$  and the contact load is  $25 \text{ N}$ .

- To calculate the specific wear rate  $k [10^{-15} \text{ N/m}^2]$  a formulae for the wear volume  $V(b, L, r)$  is to be derived as a function of the wear scar geometry.
- Calculate the specific wear rate  $k [10^{-15} \text{ N/m}^2]$ .



---

**Problem 6.8: True or untrue?**

## Wear mechanisms

1. The wear mechanism, in dynamically loaded clamping's in which only some micro movement takes place, is called fretting wear. The wear mechanism is adhesion or abrasion (page 184).
2. If the wear rate is accelerated by corrosion it is called corrosive wear, the primary wear process can be abrasive wear or adhesive wear (page 183).
3. In rolling contacts surface fatigue (sub-surface initiated) is generally the predominant wear mechanism (page 185).
4. The wear rate in dry sliding steel - yellow metal contacts is dominated by adhesive wear also called galling wear ( page 183).
5. Wear particles from a hard steel surface can be much harder than the surface hardness from which they originate.
6. In a polymer-steel contact under sliding the wear rate of the polymer will always exceed the wear rate of the much harder steel (page 184).
7. Three body abrasive wear refers to erosion, which is the predominant wear mechanism in sand-slurry pumps (page 188).
8. False Brinelling is a result of plastic deformation in concentrated contacts creating shallow indents (page 185).
9. Fretting wear typically occurs by reciprocating motion such as in a piston - liner contact (page 184).
10. Adhesive wear occurs when strong adhesive bonding between interacting asperities causes micro-welding (page 182).
11. Junctions between asperities created by adhesive wear may shear off whereby material transfers from one surface to the other and/or wear debris are created when ruptured out of the surface (page 182).
12. When two sliding bodies stick by severe galling over a large area and the bodies do not get separated on their own, this phenomenon is called seizure (page 183).
13. Scuffing is a type of adhesive wear that typically occurs in dry sliding steel contacts (page 182).
14. Scuffing, cold welding and galling are all related to adhesive wear (page 182 & 183).
15. Scuffing is a type of adhesive wear that typically occurs when the lubricant film collapses (page 182).
16. Scuffing can be prevented when using lubricants with extreme pressure EP-additives (page 182).
17. Stainless steel fasteners are sensitive for galling, which can be explained by the very thin oxide layer (page ).



18. Extreme thread galling on stainless fasteners leads to seizure also called cold welding, the actual freezing together of the threads (page 155, 183).
19. The flash temperature is defined as the contact temperature in the micro contacts formed between interacting asperities (page 174).
20. Alloyed steel is generally beneficial in preventing adhesive wear, because alloying reduces surface energy and at the same time increases hardness (small  $\gamma/H$ ) (page 153).
21. In a polymer to steel contact the steel surface might be subjected to (mild) three body abrasive wear by oxides and other particles of the steel self that embed in the polymer surface (page 184).
22. The predominant wear mechanisms that ruins the surfaces of a stainless steel air bearing pad that crashes at a hard chromed track is abrasive wear (page 182).
23. The wear mechanism in dry sliding steel - steel contacts operating in vacuum is adhesive wear, also called galling wear (page 182, 183).

#### Finishing techniques

1. Lapping is based on three body abrasion.
2. When lapping a hardened steel surface using a pearlitic cast iron disc and diamond powder in a mixture of petrol and oil, the relative soft cast iron disc doesn't suffer from wear while the hard steel surface becomes finished to a high polish.
3. A steel part can be made less sensitive to three body abrasion by making it from a lower grade steel.
4. Accumulation of work piece material on the cutting edge of a tool can be characterized by galling, a severe form of adhesion.
5. The main difference between surface grinding and super finishing techniques like polishing, lapping and honing is that super finishing is limited to the smoothening of roughness summits.
6. Lapping and Polishing are typically superfinishing techniques intended to smooth the surface roughness.
7. Burnishing is a super finishing technique based on three body abrasion.
8. Burnishing is the plastic deformation of a surface. Burnishing smears the texture of a rough surface and makes it shinier.

#### Wear rate estimation

1. Materials that show relatively low friction have high wear resistance.
2. PTFE bearings are favourable over many other polymers because of the very low friction and high wear resistance.
3. Materials are classified as hardwearing materials when  $k \leq 10^{-15} \text{ Pa}^{-1}$ .

4. The specific wear rate  $k$  in Archard's equation is a constant, which is independent of the load and velocity.
5. The dimension of the specific wear rate  $k$  is  $[m^3/Nm = m^2/N = 1/Pa]$ .
6. The wear factor  $K$  is defined as the quotient of the specific wear rate and the materials hardness. The wear factor  $K$  is dimensionless [-].
7. The wear rate of metals that are subjected to abrasive wear is in a large range of operating conditions by approximation inversely proportional to their hardness.
8. Metals with good metallurgical compatibility are less sensitive to adhesive wear.
9. Carburizing of low carbon steels creates a beneficial non-metallic surface character that makes the steel less sensitive to adhesive wear (small  $\gamma/H$ -ratio)
10. The "stationary contact" is defined as the contact surface that is stationary with the load vector.
11. The "stationary contact surface" in a sliding bearing is defined as the contact surface that is not rotating with the load vector.
12. A stationary contact will result in less wear compared to a non-stationary contact.

#### How to manage abrasive / adhesive wear

13. One of the guidelines to counter both adhesive wear and abrasive wear is to select materials with high hardness and a relative hardness of at least 80%.
14. One of the guidelines to counter adhesive wear is to select both surfaces of high material hardness.
15. Adhesive wear can effectively be reduced by carburizing one of the surfaces and nitriding the other.
16. Adhesive wear can effectively be reduced by carburizing both surfaces.
17. Adhesive wear can effectively be reduced by applying a steel to polymer contact.
18. Two polymers in relative sliding are not sensitive to adhesive wear.
19. Electro polishing prior to kolsterizing of stainless steel fasteners helps to prevent galling / seizure, infinite smooth is optimal.
20. Several metals show a threshold load at which the oxide layer is no longer capable to separate the surfaces. At this point the wear mechanism changes from abrasive to adhesive wear.

## Solutions

6.1)

- a) Sliding distance  $s = \pi d_2 n = 3.3 \text{ km}$   $h_{nut} = k p s = 0.165 \text{ mm}$   
 b) Sliding distance  $h_{spindle} = k p s A_{nut} / A_{spindle} = 0.165 / 20 \text{ mm}$

6.2)

- a) Substitution of  $F=pA$  and  $V=m/\rho$  in Archard's equation (eq. 6.1 page 191) gives  $s=53 \text{ km}$ , and  $t=245 \text{ hours} = 10.2 \text{ days}$ .  
 b) Equating the frictional heating to the heat transfer by convection gives  $Q = F_f v = h_c A dT$ , where  $F_f = \mu F$  and  $v = n s_i$ . This results in  $Q=1.44 \text{ W}$  and a temperature increase of  $dT=2.6^\circ\text{C}$ . The mass temperature is much lower than that of the human body and consequently the test frequency can be increased.

6.3)  $h = k p s$   $s = \frac{h}{k p} = 33.3 \text{ km}$

6.4) Config a) and d):  $V = d L \delta$ , config b) and c),  $V = \pi d L \delta$

- a) the wear volume in config a) and d) is  $\pi$  times smaller than in config b) and c), for the same shaft displacement.  
 b) In config a) and d) a smooth conform contact is created while in config b) and c) the conformity degrades by wear. A smooth conform contact is preferable because of the lower contact pressure with the same load and if lubricated, a lubricant film can be created more easily.

6.5)  $V = k F s$   $V = \pi d h L$   $s = 365 \cdot 24 \cdot 60 \pi d n$   $h = 0.245 \text{ mm}$   
 $P = F_f v$   $F_f = \mu F$   $v = \pi d n / 60$

6.6)

a)  $h = k p s$ ,  $p = F / (\pi 30^2)$ ,  $s = S/3$ ;  $h = 3.5 \cdot 10^{-3} \text{ mm}$ ,  $10/h \approx 2827 \text{ stops}$ .

b)  $V_{pad} = V_{disk}$ ;  $\frac{A_{disk}}{A_{pad}} = \frac{\pi (100+30)^2 - \pi (100-30)^2}{\pi 30^2} = 13.3$ ,  $h_{disk} = \frac{A_{pad}}{A_{disk}} h_{pad} = 0.75 \text{ mm}$   
 alternative method:  $h_{disk} = \frac{k F s}{A_{disk}} = 0.75 \text{ mm}$

c)  $h_{pad} = k_{pad} \frac{F}{A_{pad}} s$   $h_{disk} = k_{disk} \frac{F}{A_{disk}} s$   
 $\frac{k_{pad}}{k_{disk}} = \frac{h_{pad}}{h_{disk}} \frac{A_{pad}}{A_{disk}}$

Braking is one of the biggest strengths of a Formula One car. The brake disks of Formula 1 cars are made of a composite material reinforced with carbon fibre. The coefficient of friction between the pads and the discs can be as much as 0.6 when the brakes are up to temperature. Steel brake disks are heavier and would exhibit a higher wear rate at these high temperatures.

The temperature of an F1-brake disk varies between 400 and 1000°C. You can often see the brake discs glowing during a race. If the racing driver hits the brakes full on before the brake disks have reached a temperature of approximately 400 degrees, the disks could explode under the thermal stresses created. These stresses are the result of large temperature gradients leading to expansion which causes large stress gradients.

As the heat created in the modern Formula One brake disk is so high, there is a constant demand to find more and more cooling. In 2001, Ferrari conceived an original way of dealing with the heat problem in brakes. The brake-duct, conducting the slipstream wind along the brake disks, is equipped with a kind of turbo. This is a rotor mounted on the wheel shaft, providing additional suction to get even more air into the brake-duct. Thanks to the rotor, the brake-duct can be made smaller, benefiting the aerodynamics. The other F1 teams have now copied the idea.

In F1 the maximum dimensions of the brake disks are laid down in regulations. During qualification, relatively thin and hence light brake disks are used because the cars only need to complete 12 laps. For the race itself the thickest possible discs are fitted on the car. It means the incurred heat can be better distributed over the brake disk material. During the race a sensor continuously measures the thickness of the brake disks. The measurements will help the driver to know when he has to go easy on the brakes to make it across the finishing line. A set of disks and pads costs as much as a compact car. For every Grand Prix race each team reckons on using twenty sets of brake disks and pads per car.

### 6.7)

a) The wear volume is described by:  $V = A L$   $A = \pi r^2 \frac{\alpha}{\pi} - b(r-h)$

Substitution of:  $\sin(\alpha) = \frac{b}{r}$   $h = r - \sqrt{r^2 - b^2}$

Results in the wear volume:  $V = L r^2 \left( a \sin(b/r) - b/r \sqrt{1 - (b/r)^2} \right)$

b) Substitution of the scar geometry gives  $V = 4.3 \text{ mm}^3$ ,  $s = \pi d n = 314 \text{ m}$ ,  $k = V / (F s) = 550 \cdot 10^{-15} \text{ m}^2/\text{N}$

### 6.8)

Wear mechanisms:

True 1, 2, 3, 4, 5, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 23

Untrue 6, 7, 8, 9, 13, 20, 22

Finishing techniques:

True 1, 2, 4, 5, 6, 8

Untrue 3, 7

Wear rate:

True 3, 5, 6, 7, 9, 10, 11, 12

Untrue 1, 2, 4, 8

Managing wear:

True 13, 14, 15, 16, 17, 20

Untrue 18, 19

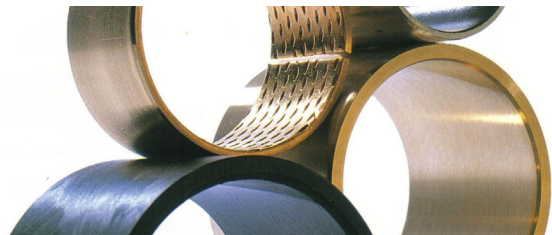
**Chapter 7 - Problems with Solutions****Problem 7.1: Design considerations plastic plain bearings**

- a) A designer is faced with the choice between a plain bearing with small shaft diameter (small  $D$ ) in combination with a long bushing (large  $L$ ) or a larger shaft diameter (large  $D$ ) in combination with a short bushing (small  $L$ ), so that the projected contact area  $LD$  is the same for both bearings. Which considerations will determine the choice? Consider the major temperature limitations.
- b) In order to obtain a larger PV-value, the designer selected a thin plastic bushing, increasing the heat dissipation to the environment via the bushing. In experiments, the LPV-value turned out to be smaller than anticipated. Explain this result.
- c) Explain why a plastic bearing loaded with high  $p$  and low  $v$  exhibit a larger LPV-value than one loaded with low  $p$  and high  $v$ .

**Problem 7.2: Preselection of materials in high temperature applications**

Consider the major temperature limitations in sliding contacts of

- metals
- plastics and resins
- ceramics and ceramic coatings

**Problem 7.3: Material selection metal plain bearings**

One guideline for material selection of boundary lubricated metallic bearings is a 3 - 5 high hardness ratio between the two mating surfaces. Explain this preferred difference in hardness.

**Problem 7.4: Material selection for concentrated contacts (Hertz)**

- a) List at least 5 failure mechanisms that can be distinguished when designing Hertzian contacts?
- b) Explain why the load capacity of Hertzian contacts formed between engineering ceramics and steel is less than between a steel to steel contact.
- c) What material selection parameter is important to select a material combination with the highest static load rating.

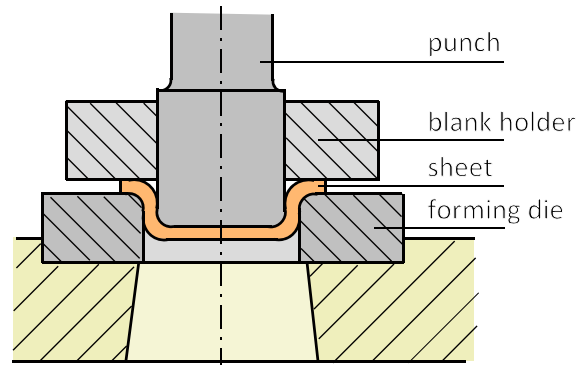
**Problem 7.5: Material selection for infinite life design**

- a) Explain why aluminium parts are never designed with stresses below the endurance strength, they are always approached with safe life design concepts.

### Problem 7.6: Deep drawing

The tool life of a drawing die strongly depends on the lubricant used, the blank material and the surface coating on the forming die.

During pressing operations, the punch and forming die are continuously in contact with new material (open system). A pin-on-disk tribometer can therefore not be used in its standard form for this application, since this implies that the pin always runs in the same track on the disk rather than come into contact with new material.



ASTM G 132-96 contains the description of a “standard test method for pin abrasion testing”. In this standard various configurations are given whereby the pin makes a specific movement over the counter surface

so as to continuously make a fresh track. The configuration of choice is one with a spherical pin moving over a steel plate, drawing parallel tracks 1 mm apart. The pin has a radius of 5 mm. The average contact stress lies in the region of the hardness of the plate material (steel plate: 64HBW or 640 MPa, aluminium plate: 36HBW or 360 MPa).

- Calculate the track width in the steel plate with a load of  $F = 150 \text{ N}$ .
- The drawing tool's steel pin is provided with a CVD-TiC coating. How can it be determined during the experiment whether the coating is worn away?
- The pin is provided with a CVD-TiC coating of  $8 \mu\text{m}$  thickness. What is the expected life of the coating when  $k = 0.1 \cdot 10^{-15} \text{ m}^2/\text{N}$ ? Express the expected life in meters track length and in the experiment duration when the sliding velocity is  $0.01 \text{ m/s}$ .
- If the friction suddenly increases, without the coating having worn out, what could have caused it?

### Problem 7.7: Tire temperature Formula 1 racing car

A racing tire performs optimally at an operation temperature of  $100^\circ\text{C}$ - $110^\circ\text{C}$ . During racing, the tire temperature is measured constantly and the data passed on to the driver. If the F1-car is getting ready for the race, the tires' temperature is raised with tire heaters. A sort of electric blanket is used to pre-heat the tires to  $80^\circ\text{C}$ . Why is the tire temperature so important and what would happen if the temperature would become too high?

### Problem 7.8: True or untrue?

- Typical abrasion effects are characterised by irregular friction with large peaks and material transfer from one surface to the other.
- Poisson's ratio is the ratio of lateral to axial strain of a material. In other words,  $\nu$  is the amount of transversal expansion divided by the amount of axial compression. For rubbers  $\nu = 0.5$  (incompressible), while for steels and polymers  $\nu < 0.5$ .
- In dry sliding metal to metal combinations the coefficient of friction is relatively large for materials with high shear strength.

4. Lubrication of metal-to-metal contacts in relative motion is generally essential to prevent severe adhesive wear that ruins the surface quality and/or to prevent very high friction.
5. Steel shafts running in combination with sinter bronze bearings impregnated with solid lubricants do not need any further lubrication until the solid lubricant is worn away.
6. In order to minimize the tendency to adhesive wear materials are selected which are metallurgical incompatible.
7. A good material selection factor for the Hertzian load capacity of a concentrated circular contact is  $(Rp0.2/E')^2$ .
8. In order to minimize the tendency to adhesive wear, materials are selected which are metallurgical compatible.
9. Bearings with lead-Babbitt layer are typically applied in hydrodynamically lubricated bearings.
10. Leaded bronze bearings are the most popular choice in machine engineering because of the good slide ability when boundary lubricated, high stiffness and good machine ability.
11. Thermosets with selected filler become more and more popular in offshore applications because of the high load capacity and good dry sliding properties with no concern of water that may enter the bearing system.
12. Sintered metals impregnated with oil running against steel are widely applied in consumer products because they are lubricated for life.
13. One of the possibilities to improve the life duration of the surface of a rolling guide is by applying a thin hard coating.
14. Thermoplastics in short plastics have advantages over metals in terms of vibration damping, low weight, inexpensive and the ability to run without lubrication.
15. Concerns about plastic bearings are the low stiffness, poor heat conductivity, large thermal expansion and large machining tolerances.
16. Examples of semi crystalline thermoplastics are POM, PA, PE-UHMW.
17. Semi-crystalline materials can be used up to some tens of degrees below the glass-transition temperature.
18. Examples of amorphous polymers are PVC, ABS, PMMA, PC.
19. Simple models to characterize visco-elasticity are the Kelvin model and the Maxwell model.
20. Visco-elasticity is modelled by a combination of Hookean springs and Newtonian dashpots, in which the spring represents elasticity and the dashpot the damping.
21. Thermoplastics in sliding motion against steel do not need to be lubricated however, lubrication may significantly reduce friction.
22. High performance plastics show high wear resistance and low friction.
23. In many self-lubricating plastics PTFE is used as a filler to achieve low friction and good wear resistance. The surface roughness of the counter surface is important to form an effective PTFE transfer film.
24. The fatigue strength of a component cyclically loaded in bending can be improved significantly by carburizing.
25. Thermosets have much higher melting temperature than thermoplastics.



26. Ceramic balls applied in deep groove ball bearings running at high speed are favourable over steel balls because of the much lower centrifugal forces.
27. Silicon Nitride balls in contact with steel are favourable over steel balls because of the higher Hertzian load capacity.
28. Thin modern hard coatings such as PVD and CVD coatings are sensitive to delamination when applied in cyclically loaded concentrated contacts.
29. A good material selection factor for the Hertzian load capacity of a concentrated line contact is  $(R_p 0.2)^2 / E'$ .
30. In many self-lubricating plastics, PTFE is used as a filler to achieve low friction and good wear resistance. Lubrication is not needed but if applied it will further improve the wear resistance. U
31. A good material selection factor for the Hertzian load from a high speed rotating ball in a ball bearing is  $(R_p 0.2)^3 / (E')^2 / \rho$  where  $\rho$  is the density.
32. A good material selection factor for a slide bearing in order to achieve a high load capacity which is limited by thermal failure is  $K dT / \mu$  where K is the heat conduction coefficient, dT the maximum contact temperature and  $\mu$  the coefficient of friction.
33. Self lubricating plastics are thermoplastics dispersed with solid lubricants such as PTFE, MoS<sub>2</sub> and graphite.
34. When lubricated with fluids, unfilled plastics show higher wear resistance than the relatively expensive self-lubricating plastics.
35. The load capacity of plastic-plastic combinations is limited by the poor heat conduction.
36. The surface durability of spur gears is calculated using Hertz theory.
37. Amorphous plastics generally exhibit higher strength, rigidity and wear resistance than semi crystalline plastics and show in general good chemical resistance to oils and grease.
38. High performance plastics, engineering plastics and general purpose plastics are classified on the basis of their stiffness.

## Solutions

## 7.1)

- a) The load capacity of plain bearings subjected to sliding is limited by the PV-value.

The failure mode (function loss) can be described as 1) loss of shaft position, 2) suddenly increase of friction and 3) jamming\*. The cause of the failures is temperature related, the maximum temperature that can be accommodated.

Cause of the failure: Frictional heating can be expressed as  $Q = \mu F v$  (Watt), or per unit of area  $Q/A = \mu p v$ . The temperature that may exist depends on frictional heating and heat dissipation to the environment. Failure occurs when the temperature of the bearing materials becomes too high.

Failure modes: Engineering plastics may fail by melting, or when the polymer becomes too weak.

\* Jamming may occur when the operational clearance becomes too small by thermal expansion during operation. The PV-value is not a limit for this phenomenon (Problem 5.13).

- b) The contact patch created by plastic deformation becomes smaller with a thin wall thickness of the bearing bushing, resulting in a larger real contact pressure and increased frictional heating per unit of surface area.
- c) The bearing running at low velocity and high contact pressure will show a larger pv-value because the coefficient of friction in plastic bearings is decreased at higher contact pressure. Furthermore, with a high load at low velocity, the contact patch created by elastic deflection of the polymer bushing by the shaft load is larger.

## 7.2)

Although metals in general can sustain high temperatures and have a good heat conduction, it is the lubricant that is needed in metal-metal contacts in sliding motion that may fail. Lubricating oils can be applied up to 150°C, however the oxidation stability of the lubricating oil may limit the exposure time to this temperature. Dry/solid lubricants, for example MoS<sub>2</sub> based, can be used with contact temperatures up to 400°C, in vacuum much higher.

Some high-performance polymers are suitable for use at high temperatures up to 150...300°C, but still have relatively low compressive strength at higher temperatures and poor heat conductivity.

Thermoset (resins) bearings may fail when the decomposition temperature is reached. It is the temperature at which the substance chemically decomposes. When the surface gets burned the wear rate will increase significantly and the shaft position will get lost.

Ceramics are very hard and can withstand high temperatures up to 400°C, but are brittle, cannot accommodate edge pressures, show high friction coefficients in combination with steel ( $\mu=0.2-0.8$ ) and are also poor heat conductors. Ceramic bearings are favourable in abrasive environments, because of the superior resistance to three body abrasive wear.

Carbon based ceramic coatings (DLC) have a good thermal conductivity when applied on a hard steel substrate. Since DLC coatings require a hard substrate edge pressures cannot be accommodated. The very low surface energy makes the DLC coating very suitable to perform at high temperatures and in vacuum where lubricants are banned.

## 7.3)

In Table 7.2 page 208 some common bearing materials are listed, ranged by the material hardness. The hardness of the counter material, in general the shaft, should be approximately 3 times harder than the hardness of the bearing material.

- Abrasion of the hard counter material is limited to polishing wear rather than scratching.
- Improved embed ability of hard abrasive particles in the relative softer bearing material.
- Edge pressures from misalignment or shaft deflections are averaged out by plastic deformation of the relative soft bearing material.

#### 7.4)

- a) Static load rating (Brinelling), Dynamic load rating for pure rolling (Fatigue, pitting, flaking, spalling), Rolling with traction (surface initiated crack), Frictional heating (Heathcote slip, cage slip, macro slip when not preloaded), Lubricant failure (EHL, thermal), Wear (False brinelling) etc.
- b) The limiting factor for the static load rating is the maximum contact pressure (material shear stress) that can be resisted. In a ceramic to steel contact the limitation is set by the maximum contact pressure of the steel. The larger rigidity of the ceramic body will cause less flattening, resulting in a more concentrated higher contact pressure.
- c) The material selection parameter for the static load rating of a point and elliptic contact is  $R_p 0.2^3 / E'^2$  (eq 4.19 page 109), for a line contact  $R_p 0.2^2 / E'$  (eq. 4.26 page 110).

#### 7.5)

- a) The problem with aluminum is, that it doesn't have a typical 'fatigue limit'. This is different from steel which has some distinct endurance limit. Aluminium will always fail if tested to a sufficient number of cycles. Therefore, the fatigue strength of aluminum is usually reported as the stress level it can survive at a large total number of cycles, usually  $5 \cdot 10^8$  cycles.

#### 7.6)

- a) With (eq. 4.10 page 105) it can be established that the load  $F/F_1 > 200$  and the deformation is consequently plastic. With this information it follows that the track width is  $b=2r$  from  $H = F/\pi r^2$ . The track width turns out to be approximately 0.55 mm.
- b) The friction in the contact between the ceramic coating and the steel is lower than when the steel of the tool comes into contact with the steel plate. Through adhesion, the friction and wear will suddenly increase sharply. Another method suitable for electrical insulating coatings is the measurement of electrical conduction.
- c) In Case 3.2 page 193, an example is given for the wear volume of a ball as a function of the flattening  $h$ . With  $h = 8 \mu\text{m}$  follows for the wear volume  $V = 1 \cdot 10^{-3} \text{ mm}^3$ . Substitution in (eq. 6.1 page 191) gives a track length of  $s = 67 \text{ m}$ . With a sliding velocity of 0.01 m/s the expected tool life would be approximately 110 minutes.
- d) Through the adhesive transfer of material the friction suddenly increases.

#### 7.7)

Figure 7.7 page 213 shows that the friction coefficient increases with temperature until a maximal value is reached; above that value the friction coefficient falls back. In order to achieve maximum traction, an operating temperature is needed which ensures a maximum friction coefficient. When the maximum friction coefficient is exceeded, fading occurs whereby the friction strongly decreases and a high level of melting takes place. Fading can also occur in the brakes, i.e. 'brake fade'. A Formula 1 car has no anti-lock braking system (ABS), which means it is possible that a racing driver ends up locking a wheel when he brakes. It extends the braking distance and the tire will undergo terrible local wear, resulting in a "flat spot".

During the formation lap, most drivers zigzag along the track to warm up the tires. At the end of this lap, the drivers park their cars at the start position and start the race a few seconds later. Tires that have reached the right temperature become very sticky, resulting in a considerable

amount of rubber pick-up from other tires which in the course of the race end up immediately next to the ideal line on the track. Whoever takes the ideal racing line experiences few problems with this. But a driver who tactically makes way for a faster colleague regularly, encounters problems because of this. It takes a few laps before the pieces of picked up rubber have cleared again from the tires.

Wet Weather tires do not come to temperature in cold water. To compensate, they are made from a softer tire compound than dry weather tires. Furthermore, the tires differ because of the groove profile which dissipates water sideways. Qualifier tires are very special tires of a super soft compound with a life duration of one or two laps. At one time, this sort of tire was used to achieve a super qualifying lap time.

During the race, pitstops can be made for the following reasons:

- to change tires (according to plan, this happens once or several times per race, or more frequently in changeable weather)
- repairs (always unexpected) or adjustments to front and/or rear wings (always unexpected)

To change the tires, a small army of at least fifteen fitters is needed to return the car to the race within 4 seconds: two fitters for the jacks at front and back, one for the 'lollipop' plus three fitters per tire (number one undoes the central wheel nut, number two removes the old tire and number three fits the new one. The 'lollipop man' (literally, from lolly) ensures with a board on a stick that the driver knows what he has to do during the stop (e.g. 'BRAKE' and 'FIRST GEAR').

**7.8)** True: 3 - 4 - 5 - 6 - 9 - 10 - 11 - 12 - 14 - 15 - 16 - 18 - 19 - 20 - 21 - 23 - 24 - 26 - 28 - 29 - 31 - 32 - 33 - 34 - 35 - 36

## Chapter 8 - Problems with Solutions

### Problem 8.1: Lubrication regimes

The slide surface of a cam-follower mechanism is partially submerged in an oil bath. After one year's operation, no wear can be observed. The finishing grooves on the slide surface are still visible.

- Explain this phenomenon.
- If the lubrication regime is not changed, could failure eventually occur?

### Problem 8.2: EP-additives

- Describe the way/when EP-additives work.
- What type of wear can be prevented with EP-additives?
- What does EP stand for and why is this nomenclature disputable?

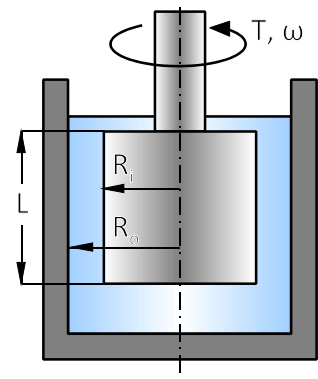
### Problem 8.3: Cylinder viscometer

A standard viscosity meter (ASTM) is the rotational cylindrical viscometer as shown in the figure. The outer cylinder is fixed while the inner cylinder is rotating at a constant angular speed of  $\omega$  by applying a torque  $T$ . Derive an equation to calculate the absolute viscosity  $\eta$  [Pa.s].

Calculate the viscosity value  $\eta$  [Pa.s] when  $\omega = 55$  rev/min,  $T = 5$  mN-m,  $L = 50$  mm,  $R_i = 10$  mm and  $R_o = 12$  mm.

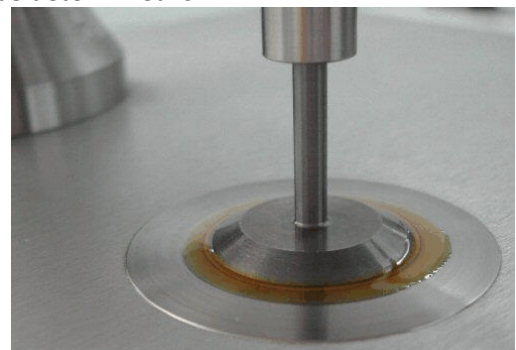
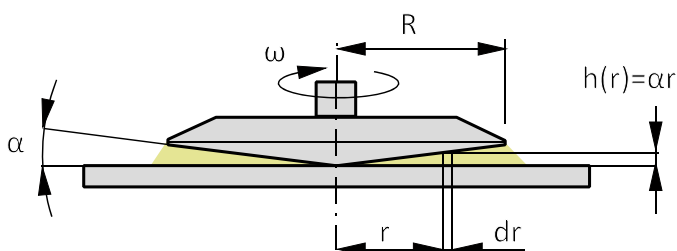
Approach:

The liquid is a Newtonian fluid. The velocity profile between the container and the rotating cylinder is linear. Neglect the end effects and bottom surface of the cylinder.



### Problem 8.4: Cone on plate viscometer

The cone on plate viscometer is the most popular method for gaining viscosity information at specific shear rates. Factors such as small sample size and ease of cleaning have helped make the cone and plate approach so popular. The oil for which the viscosity needs to be determined is placed between the cone and the plate.



Before the measurement can take place, some time is required to bring the cone, which is at the same temperature as the plane, to the required temperature. To determine the driving torque as a function of the viscosity, the following questions need to be answered.

- a) Derive an equation for the shear rate  $\partial u / \partial z$  at distance  $r$  from the centre.
- b) Derive an equation for the viscous shear force  $F_t(r)$  of element  $dr$ .
- c) Derive an equation for the viscous friction torque  $M$ .

---

#### Problem 8.5: Special lubricants

List a few specific functional requirements of lubricants for

- a) Gear lubrication
- b) Hydraulic oils
- c) Cutting oils.

---

#### Problem 8.6: True or untrue?

1. Elasto-hydrodynamically lubricated (EHL-) bearings are not subjected to mechanical wear / the roughness of interacting surfaces.
2. Standard rolling bearings that have failed finally by surface fatigue have run during their life in the elastohydrodynamic lubrication (EHL-) regime.
3. Polymer - metal contacts do not need to be lubricated, but lubrication can help to reduce friction and wear.
4. Dry sliding and boundary lubrication (BL) are similar, the lubricant if applied is not effective in reducing friction.
5. Polymer - steel contacts are very well lubricated by silicon oil. Since silicon oil doesn't adhere very well to the metal surface it is less effective in steel-steel contacts than mineral oils.
6. Vegetable oils adhere very strong to metal surfaces which make them superior to mineral oils and synthetic oils, when operating in the BL-regime.
7. Compounded oils consist of high viscous (ISO VG 460, 680 or 1000) mineral oils with 3-10% fatty oil (from animal or vegetable oil).
8. Compounded oils are superior in high temperature applications because of their oxidation stability.
9. Vegetable oils are the best choice when cutting thread in a tap hole for example. This because of the environmental compatibility and extreme good lubricity.
10. The coefficient of friction in the hydrodynamic lubrication (HL-) regime is typically 0.001, in the boundary lubrication regime 0.1.
11. The coefficient of friction in the mixed lubrication (ML-) regime decreases with the velocity and increases with the load.
12. HL lubrication can be created only with relative high surface velocity.
13. Lubricants developed for boundary lubrication have superior lubricity (anti wear and anti friction properties).
14. Animal or vegetable oils are per definition classified as bio-lubricants.
15. Synthetic esters are good bio-lubricants, mineral oils not.
16. The viscosity index (VI) of an automotive lubricant is not less important than the lubricity since 80% of the total engine losses come from viscous shear.
17. Extreme Pressure (EP-) additives are applied to generate some extra pressure in the HL-regime.
18. EP-additives can be harmful, if too aggressive they may cause serious corrosive wear in the BL-regime.
19. Oils with a high viscosity value easily flow through a capillary.
20. Crystalline polymers have superior wear resistance compared to amorphous polymers.

## Solutions

## 8.1)

- a) As a result of elastohydrodynamic lubrication the surfaces do not make contact.
- b) Through the elastic deformation that is likely to occur in a concentrated contact, surface fatigue may eventually set in.

## 8.2)

- a) The friction between the asperity summits creates a high (flash) temperature. Under the influence of the high temperature, the EP additives form a chemical protective oxide layer with high shearing strength which can prevent pure metal contact.
- b) Without this protective layer, the asperity summits may be welded together and then break apart again with continuous movement (scuffing).
- c) EP stands for Extreme Pressure, whereas the chemical reaction establishes itself through the Extreme Temperature (flash temperature) between intermeshing asperity summits.

## 8.3)

$$a) \tau = -\eta \frac{du}{dr} \quad u(R_i) = \omega R_i \quad u(R_o) = 0$$

The negative sign is inserted to indicate that  $u$  decreases as  $r$  increases.

$$\text{The shear stress can be written as: } F(r) = \tau 2\pi r L \quad T(r) = \tau 2\pi r^2 L \quad \tau = \frac{T}{2\pi r^2 L} = -\eta \frac{du}{dr}$$

$$\text{Rearranging gives: } \frac{T}{2\pi L} \int_{R_i}^{R_o} \frac{1}{r^2} dr = -\eta \int_{\omega R_i}^0 du \quad \frac{T}{2\pi L} \left( \frac{1}{R_i} - \frac{1}{R_o} \right) = \eta \omega R_i \quad \eta = \frac{T(R_o - R_i)}{2\pi L \omega R_i^2 R_o}$$

Substitution of the numerical value gives  $\eta = 0.5 \text{ Pa}\cdot\text{s}$ .

If the gap between the wall and cylinder is small, then  $R_o - R_i \Rightarrow dR \text{ gap}$ , and  $R_i^2 R_o \Rightarrow R^3$  where  $R$  is the mean radius.

## 8.4)

$$a) \tau = \eta \frac{\partial u}{\partial z} \quad u(r) = \omega r \quad h(r) = \alpha r \quad \frac{\partial u}{\partial z} = \frac{\omega r}{\alpha r} = \frac{\omega}{\alpha}$$

It is found that a uniform shear rate is generated across the entire sample for any given rotational speed.

$$b) F_f(r) = \tau 2\pi r dr, \quad \tau = \eta \omega / \alpha$$

$$c) M = \int_0^R r \tau 2\pi r dr = \int_0^R r \eta \frac{\omega}{\alpha} 2\pi r dr = \frac{2\pi}{3} \frac{\eta \omega R^3}{\alpha}$$

A doubling of the cone radius  $R$  results in a spindle torque increase of a factor eight. This is of great benefit when measuring low viscosity products where an instrument's minimum measurable torque is limited.



**8.5)**

- a) Gear lubricants: extreme pressure additives to prevent scuffing, high pressure coefficient to enable elasto-hydrodynamic lubrication, anti foaming.
- b) Hydraulic oil, corrosion resistance, biodegradable.
- c) Cutting oil: great lubricity, extreme pressure (EP), cooling, corrosion, compatibility.

**8.6)** True: 1 - 2 - 3 - 5 - 6 - 7 - 9 - 10 - 11 - 13 - 14 - 15 - 16 - 18 - 20.

Always read the statements carefully. With one word changed the statement that was true can become untrue. So, don't recognize a statement but understand the statement.

## Chapter 9 - Problems with Solutions

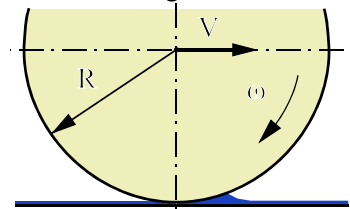
### Problem 9.1: Reynolds' equation

Which assumptions have been made in deriving the Reynolds equation?

### Problem 9.2: Aquaplaning

A thin water film develops between a car tire and the road surface. Because of the water film, the traction forces between the tire and the road surface are almost lost.

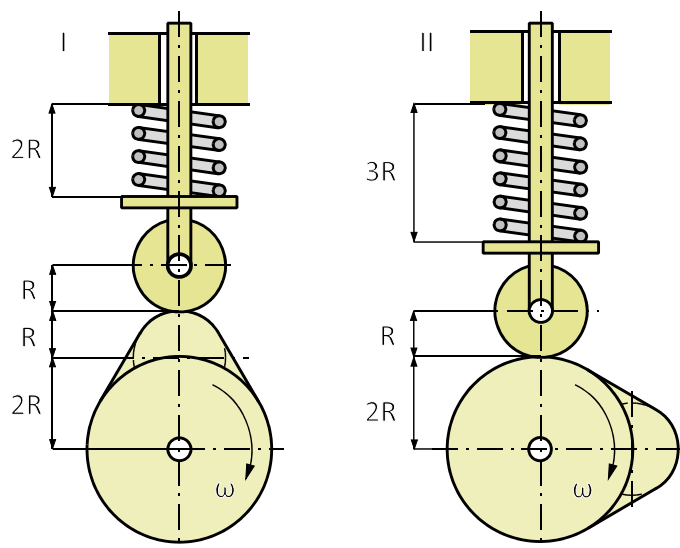
- What is the effective surface velocity that causes aquaplaning? Assume that there is no slip yet ( $V = \omega R$ ).
- Several measures are considered to make contact with the road surface. Indicate to which extent these actions make sense by considering the effective surface velocity in the following instances:
  - pushing the gas pedal ( $\omega R = 10V$ ),
  - active braking ( $\omega R = 0$ ),
  - putting the car into reverse gear ( $\omega R = -10V$ ),
  - pushing in the clutch ( $0 \leq \omega R \leq V$ ).



### Problem 9.3: Cam-follower mechanism

For an optimal lifetime performance and reliability of a cam-follower mechanism, EHL is required between the cam and the rotating follower. The minimum film thickness in a line contact can be calculated using (eq. 4.51 page 124).

- Demonstrate in which position of the cam - I or II - the EHL will fail first if no slip occurs and when the angular velocity of the cam is constant.
- see a) but in the event of 100% slip.
- The film thickness  $h_c$  to be calculated gives an indication if EHL may occur. Where does the minimum film thickness for EHL depends on?



**Problem 9.4: Grinding of rollers**

A large rotating grinding stone is used with plain hydrodynamic journal bearings. (Rolling bearings generally display high frequency vibrations which compromise the accuracy of movement and hence the surface quality of the grinding). To improve the surface quality, the rotational velocity a grinding stone is increased. At this high rotational velocity the grinding stone starts to vibrate at a frequency equalling half the rotational velocity.

- What will be the origin of this vibration?
- What measures can be taken to prevent the vibration without sacrificing any of the rotational speed?
- Someone suggests replacing the hydrodynamic bearing of the grinding stone by a hydrostatic bearing. Is this a good advice?

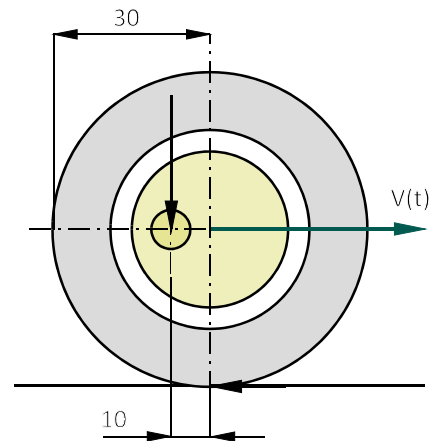
**Problem 9.5: Circumferentially grooved hydrodynamic bearing**

Sometimes bearings are used that contain a central circumferential lubricant feed groove. Examine by how much the load capacity of this type of bearing decreases as a result of the groove. Use the dimensions of the bearing given in Case 9.3a. Consider the circumferentially grooved bearing as two bearings with  $L/D=0.5$ .

**Problem 9.6: Squeezing Disk brake**

Before the brake shoes of a disk brake make contact with the brake disks some rain water will have to be dispersed. The time required to do this and the factors influencing the time will be examined here.

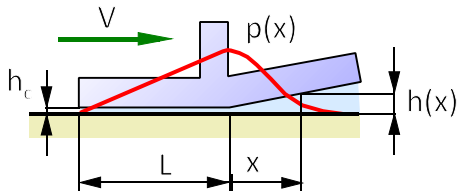
The geometry of a brake shoe is approximated by a round disk with a radius of  $R=40$  mm. The viscosity of water is  $\eta=0.001$  Pa s. The squeeze force amounts to approximately 6 kN. Because of inaccuracies in shape and roughness of the slide surfaces, mechanical contact already exists at  $h_2=3$   $\mu\text{m}$ .



- Explain why this problem can be considered a “pure” squeezing film.
- Calculate the time delay as a result of the squeeze effect and the relative increase in brake distance resulting from the squeeze effect, if the driving speed is 100 km/h.
- How can this time delay be reduced?
- To limit the wear on the brake shoes and brake disks, manufacturing the brake disks with a hard wearing material and finish them very smoothly is suggested. What do you think of this proposal?

**Problem 9.7: Tube expansion**

Consider a linear guide system using linear ball bearings (also called ball bushings). These bearings are factory pre-lubricated with grease. Due to the applied amount of grease and the use of double-lip seals (2LS), many applications will not require relubrication because in these cases the theoretical relubrication interval exceeds the bearing life.



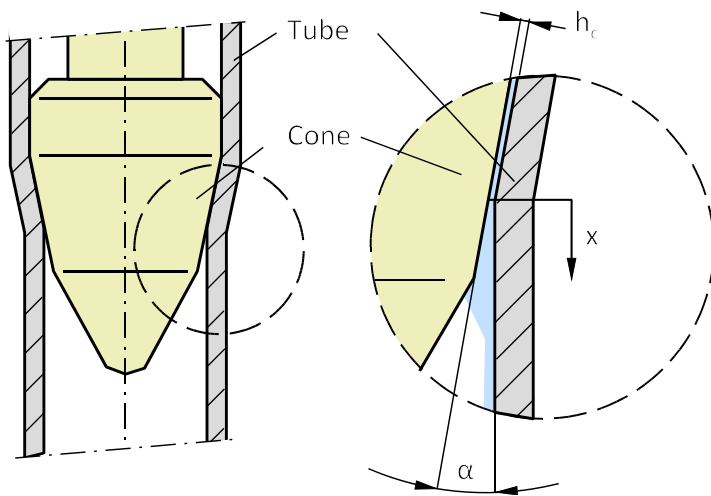
The axial movement results in a hydrodynamic pressure build-up and will bend the lip upwards creating a wedge shaped region with film thickness  $h(x)$ . The film height over the length  $B$  is approximately uniform. The flow in this region is dominated by a Couette flow.

Consider a wedge shaped region with film thickness  $h(x)=h_c+\alpha x$  where  $\alpha$  [rad] is the angle of inclination. Derive an equation for the film thickness  $h_c$ .

### Problem 9.8: Tube expansion

To expand the diameter of a tube a tapered cone is forced through the bore. The objective is to maintain a thin lubrication film between the pipe and the tapered cone. Consider only the wedge effect and determine the parameters that will influence film thickness  $h_c$ .

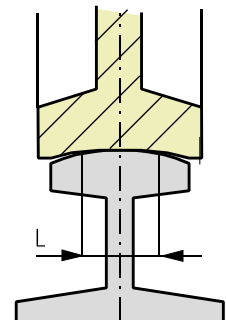
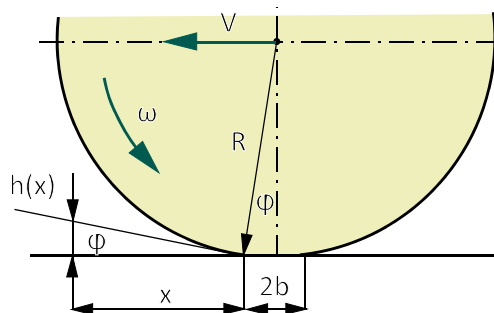
<https://www.youtube.com/watch?v=B0nF7-V-d6Q>



- Explain why a parallel film will be created.
- Derive an equation for  $dp/dx$  and  $p(x)$
- Convert the equation for  $p(x)$  to an equation with the film thickness  $h_c$ .
- Is the iso-viscous solution realistic?

### Problem 9.9: Rail-wheel contact

- In the wedge-shaped inlet region of a rail wheel contact, by approximation  $h(x)=\phi x$  ( $\phi$  in radians) applies. Derive on the basis of geometric considerations the value of  $\phi$  (consider  $\phi$  to be small).



- Heavy rainfall has created a thin water film. The wheel and the rail remain in contact (blocked film). Derive an equation for the pressure gradient in the inlet region, considering a one-dimensional flow and  $h(x)=\phi x$ . There is no slip.

- c) Why will the pressure in reality not infinitely increase when  $x=0$  ?
- d) Assume that a thin parallel film is generated. Draw a possible actual hydrodynamic film pressure distribution (inlet and conjunction area).
- e) Now derive an equation for the pressure gradient in the inlet area by equating the flow in the inlet area with the flow in the conjunction area.
- f) Calculate the pressure  $p(x=0)$ . The film thickness in the inlet region is now  $h(x)=h_0+\phi x$ .
- g) Consider that the load capacity of the film is almost entirely determined by the pressure in the conjunction area. What, then, is the load capacity?

## Solutions

- 9.1) Laminar flow, Newtonian fluid, predominated viscous shear forces, iso-viscous and constant pressure across the film, no slip condition, Euler's coordinate system.

## 9.2)

- a) Aquaplaning  $U_e = V + \omega R = 2V$ .  
 b) pushing the throttle wide open  $|U_e| = 11V$ , active braking  $|U_e| = V$ , switching to reverse gear  $|U_e| = 9V$ , holding down clutch  $V < |U_e| < 2V$ . Conclusion: braking or pushing down the clutch, preference for pushing in the clutch to gain speed again when contact is re-established.

## 9.3) Cam-follower mechanism

- a) Position I: Both contact surfaces move along with the film profile at equal velocity ( $U_1 = U_2 = U_F = 3\omega R$ ), so that no lubricant can be dragged into the converging wedge:  $U = U_1 + U_2 - 2U_F = 0$ .

Position II: The film profile is stationary:  $U_F = 0$ . Without slip, both contact surfaces drag the lubricant along as  $U_1 = U_2 = 2\omega R$  into the converging wedge:  $U = U_1 + U_2 = 4\omega R$

Conclusion: With a rotating follower, EHL will fail first in position 1.

- b) Position I: The film profile and the cam surface move to the right with  $U_1 = U_F = 3\omega R$ . Let the coordinate system move along again with the film profile:  $U = U_1 - 2U_F = -3\omega R$

Position II: The film profile and the follower stand still  $U = U_1 = 2\omega R$

In position I, the effective sliding velocity is greatest. However, the contact geometry in this position is less favourable. The film thickness will now be established more accurately for both positions.

$$\frac{(h_c)_1}{(h_c)_2} = \left(\frac{3}{2}\right)^{8/11} \left(\frac{1/2}{2/3}\right)^{2/11} \left(\frac{2}{1}\right)^{-1/11} = 1.34 \cdot 0.90 \cdot 0.94 = 1.14$$

Conclusion: With the stationary follower, EHL will fail first in position II,  $(h_c)_1 > (h_c)_2$ .

- c) The required value of  $h_c$  can be derived from the roughness of both surfaces.

## 9.4)

- a) The effect of cavitation is limited because of the small eccentricity, resulting in the load vector and the deflection vector being almost perpendicular to each other. This results in unstable behaviour, in this case at half the rotation speed, called half-omega whirl.  
 b) By choosing a larger bearing clearance the eccentricity will increase, which means the operational speed can be maintained at the higher level.  
 c) With a hydrostatic pressure in the bearing cavitation is suppressed. If any additional hydrodynamic pressure can build up, the system becomes unstable. A large plain bearing area that enables the build up of hydrodynamic pressure must therefore be avoided.

## 9.5)

The Sommerfeld number of a bearing with  $\epsilon=0.15$  and  $L/D=1$  amounts to  $S=9.12$ , with  $L/D=0.5$  it will be  $\Phi=5.08$  (Program 8.2). The load capacity of the bearing is proportional with the product of the Sommerfeld number and the bearing length. This results in the quotient:

$F(L/D=0.5)/F(L/D=1)=(5.08/9.12)(0.5/1.0)=0.28$ , i.e. the load capacity of a bearing with  $L/D=0.5$  is 28% of a bearing with  $L/D=1.0$ . For the circumferential grooved bearing, it follows that the load capacity will amount to  $2 \cdot 28\%=56\%$  of the bearing having an axial groove.

## 9.6) Disk-brake

- With a parallel film and in the case of the absence of a wedge-shaped inlet region, no wedge effect will occur.
- Substitution of  $l=F \cdot t$  in ( ) and assuming  $(h_2/h_1)^2 < 1$  gives  $t=0.12$  sec,  $s=v \cdot t=3.3$ m.
- The time delay due to the squeeze effect can be reduced by providing grooves into the brake shoes.
- A smooth finish reduces the film thickness  $h_2$  with the result that the squeeze time  $t \sim 1/h_2^2$  will strongly increase. Halving the film thickness at which contact take place will produce a squeeze time that is four times as large.

## 9.7) Reciprocating seal

$$-\frac{h_3}{12\eta} + \frac{Uh}{2} = \frac{Uh_c}{2} \quad \frac{dp}{dx} = 6\eta U \frac{h-h_c}{h^3} \quad p(x) = 6\eta U \int_0^x \frac{h-h_c}{h^3} dx, \quad h=h_0+\phi x$$

$$p(x) = 6\eta U \int \frac{1}{(h+\phi x)^2} - \frac{h_c}{(h_c+\phi x)^3} dx \quad p(x) = \frac{6\eta U}{\phi} \left( -\frac{1}{(h+\phi x)^2} + \frac{1}{2} \frac{h_c}{(h_c+\phi x)^2} \right) + C$$

$$\left. \begin{array}{l} p(\infty)=0 \rightarrow C=0 \\ p(0)=p_{\max}, \quad p_m=p_{\max}/2 \\ U=V \end{array} \right\} \quad h_c = \frac{6\eta V}{\phi p_m}$$

## 9.8) Tube expansion

- Through plastic deformation the pressure in the film equals the extrusion pressure of the tube. With the uniform pressure  $p=p_e$ ,  $dp/dx=0$  then  $q_x=Uh_c/2$  where  $h_c$  is the uniform Couette film thickness.
- The iso-viscous solution method for a wedge shaped inlet and uniform interface gives

$$-\frac{h_3}{12\eta} + \frac{Uh}{2} = \frac{Uh_c}{2} \quad \frac{dp}{dx} = 6\eta U \frac{h-h_c}{h^3} \quad p(x) = 6\eta U \int_0^x \frac{h-h_c}{h^3} dx, \quad h=h_0+\phi x$$

$$p(x) = 6\eta U \int \frac{1}{(h+\phi x)^2} - \frac{h_c}{(h_c+\phi x)^3} dx \quad p(x) = \frac{6\eta U}{\phi} \left( -\frac{1}{(h+\phi x)^2} + \frac{1}{2} \frac{h_c}{(h_c+\phi x)^2} \right) + C$$

$$\left. \begin{array}{l} p(\infty)=0 \rightarrow C=0 \\ p(0)=p_{\max} \\ U=V \end{array} \right\} \quad h_c = \frac{3\eta V}{\phi p_{\max}}$$

Where the upper limit of  $p_{\max}$  equals the tensile strength of the steel tubing.

- Viscosity increase by the pressure-viscosity coefficient and decrease by frictional heating is not considered in the solution above.



### 9.8) Rail-wheel contact

- a)  $R\phi=b$ ,  $\phi=b/R$ .
- b) In a wedge shaped film with  $h(x=0)=0$  the pressure gradient  $dp/dx$  becomes infinite.
- c) The calculation does not take into account surface roughness, sideways flow and EHL.
- d) This solution method is shown in (?).
- e) Substitution of  $U_e=-2\omega R$  and  $\phi=b/R$  in (?) results in  $p_c$

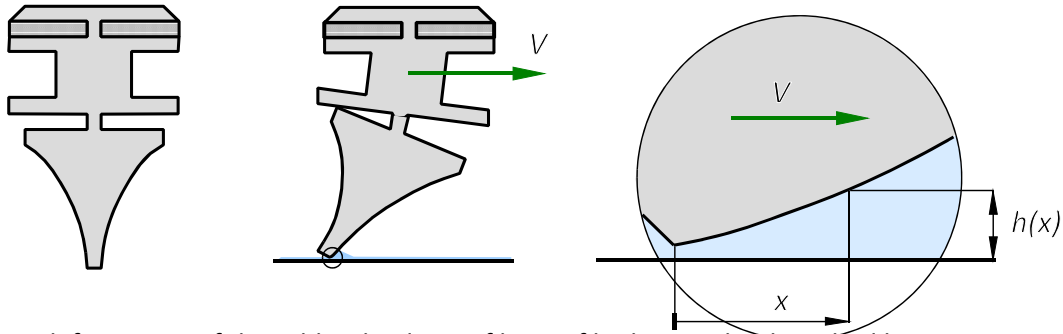
$$p_c = -\frac{3\eta U}{\phi h_0}, \quad \phi = \frac{b}{R}, \quad U = -2\omega R; \quad p_c = \frac{6\eta\omega R^2}{b h_0}$$

- f)  $F = p_c 2bL$

## Chapter 10 - Problems with Solutions

## Problem 10.1: Windscreen wiper

Hydrodynamic lubrication takes place between a windscreen wiper and the windscreen of a car. Without hydrodynamic lubrication, when the windscreen is dry, the wiper will move in jolts over the windscreen due to stick-slip. With hydrodynamic lubrication, the very thin layer of water between the wiper and the window will evaporate shortly after the wiper has passed over it.



The elastic deformation of the rubber leads to a film profile that can be described by an exponential function,  $h(x) = h_0 e^{\alpha x}$  with  $\alpha = 20.000 \text{ m}^{-1}$ . The film cavitates in  $x=0$ . The press-on force of the 40cm long wiper blade amounts to 6 N, the velocity of the wiper  $V = 1 \text{ m/s}$ , the viscosity of water is  $\eta = 0.001 \text{ Pa}\cdot\text{s}$ .

- Give an equation for the pressure gradient with the integration constant  $h_c$ . ( $h_c$  is the film thickness in the location of  $dp/dx=0$ ).
- Derive from the equation of the pressure gradient  $dp/dx$  the pressure distribution  $p(x)$ .
- Solve the relationship between the constant of integration  $h_c$  and  $h_0$ .
- Derive an equation for the load capacity per unit of length.
- Calculate the minimal film thickness between the windscreen wiper and the window?
- Calculate the thickness of the water layer that will evaporate immediately after the wiper has passed?
- Calculate the maximum value of the hydrodynamic pressure  $p_{\max}$  that will occur.

## Solutions

## 10.1) Windscreen wiper

$$a) \quad \frac{dp}{dx} = 6\eta U \frac{h-h_c}{h^3} \quad p(x) = 6\eta U \int_0^x \frac{h-h_c}{h^3} dx, \quad h = h_0 e^{\alpha x}$$

$$b) \quad \int_0^x \frac{1}{(h_0 e^{\alpha x})^2} dx = \frac{1}{h_0^2} \int_0^x e^{-2\alpha x} dx = -\frac{1}{2\alpha h_0^2} (e^{-2\alpha x} - 1)$$

$$\int_0^x \frac{1}{(h_0 e^{\alpha x})^3} dx = \frac{1}{h_0^3} \int_0^x e^{-3\alpha x} dx = -\frac{1}{3\alpha h_0^3} (e^{-3\alpha x} - 1)$$

$$p(x) = 6\eta U \left( -\frac{1}{2\alpha h_0^2} (e^{-2\alpha x} - 1) + \frac{h_c}{3\alpha h_0^3} (e^{-3\alpha x} - 1) \right)$$

$$c) \quad p(\infty) = 0: \quad 0 = 6\eta U \left( -\frac{1}{2\alpha h_0^2} (0 - 1) + \frac{h_c}{3\alpha h_0^3} (0 - 1) \right), \quad h_c = \frac{3}{2} h_0$$

$$d) \quad \frac{F}{L} = \int_0^\infty p(x) dx = \frac{6\eta U}{2\alpha h_0^2} \left( -\frac{e^{-2\alpha x}}{-2\alpha} + \frac{e^{-3\alpha x}}{-3\alpha} \right) \Bigg|_0^\infty = -\frac{1}{2} \frac{\eta U}{\alpha^2 h_0^2}$$

$$e) \quad F = 6N, \quad L = 0.4m, \quad \alpha = 20.000 m^{-1}, \quad \eta = 0.001 \text{ Ns/m}^2, \quad U_e = -1m/s$$

$$h_0 = \frac{1}{\alpha} \sqrt{\frac{\eta U L}{2F}} = 0.29 \cdot 10^{-6} m$$

$$f) \quad h_c = 3/2 h_0$$

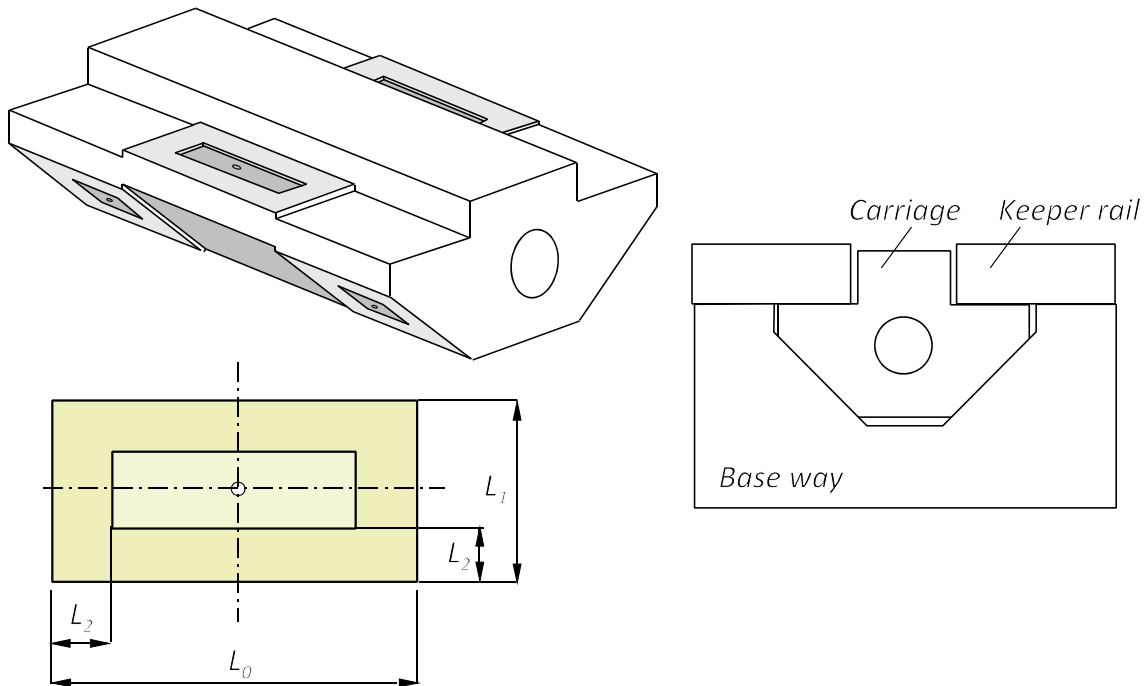
$$g) \quad h_c = h_0 e^{\alpha x}, \quad e^{\alpha x} = h_c/h_0, \quad x = \ln(h_c/h_0)/\alpha, \quad x \approx 20 \cdot 10^{-6} m$$

$$p_{\max} = p(20 \cdot 10^{-6}) = 2025 Pa$$

## Chapter 11 - Problems with Solutions

## Problem 11.1: Linear axis with hydrostatic bearings

Consider one of the two linear axis of a high precision lathe shown below. The dimensions of the bearings on the top surface are  $L_0 = 50$  mm,  $L_1 = 30$  mm and  $L_2 = 10$  mm. The gauge pressure of the pump is  $p_s - p_a = 4$  MPa. The pressure factor  $\beta = (p_r - p_a)/(p_s - p_a) = 0.5$ . Consider a linear pressure drop from  $p_r$  to  $p_a$ . The film thickness at nominal load  $F(h=h_0)$  is  $h_0 = 15 \cdot 10^{-6}$  m. The pocket depth  $h_2 = 20 h_0$ . The slider moves with  $v=0.1$  m/s to it's target point. The viscosity of the lubricant is  $\eta = 0.15$  Pa-s, the density  $\rho = 950$  kg/m<sup>3</sup> the specific heat capacity  $c=2000$  J/(kg-K).

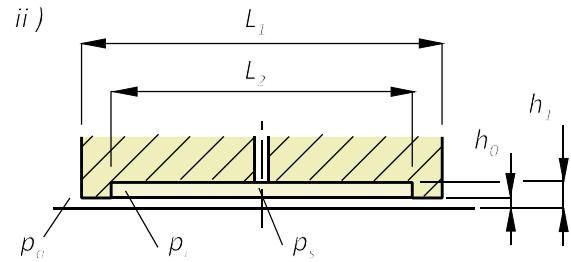
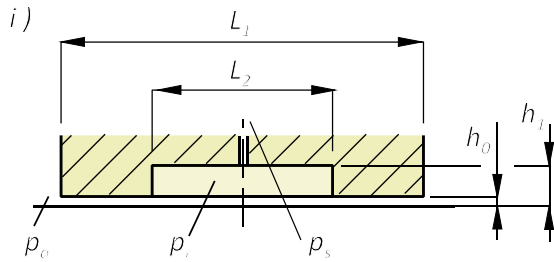


- Calculate the effective contact area  $A_e/A$ .  $A_e$  is defined as  $F = (p_r - p_a)A_e$  where  $F$  is the bearing load,  $p_r - p_a$  is the gauge pressure in the pocket.
- Calculate the flow  $Q$  [ litre / hr]
- Calculate the length of the restrictor when assuming a viscous flow and check whether the flow in the restrictor remains laminar ( $Re < 2300$ ). The diameter of the restrictor is  $d = 0.15$  mm.
- Calculate the viscous shear force  $F_f$  [N] of the individual bearing elements.
- Calculate the temperature increase of the lubricant by viscous shear.

Hint: The pumping power  $N = (p_s - p_a)Q$ ,  $N = \dot{m} c dT$ ,  $\dot{m} = \rho Q$  which gives:  $p_s - p_a = \rho c dT$ . Note that the flow rate  $Q$  is cancelled out.

**Problem 11.2: E.P. bearings with shallow pocket**

A choice must be made between one of two types of long rectangular hydrostatic thrust bearings.

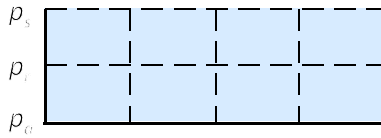


- i) a hydrostatic bearing with external restrictor. The pocket depth in this bearing is more than 10 times larger than the film thickness  $h_0$ .
- ii) a hydrostatic bearing with shallow pocket. The pocket depth in this bearing is approximately the same as the film thickness  $h_0$ .

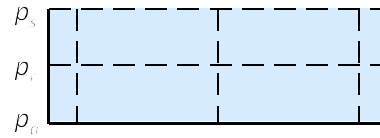
The length of the bearing is that large that a one-dimensional flow can be considered. With nominal load, the pressure factor  $\beta_0(h_0)=0.5$ . Answer the questions below for both bearing configurations.

- a) Draw the pressure distribution in both bearings.

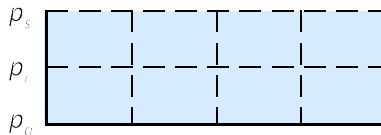
i) with film thickness  $h_0$



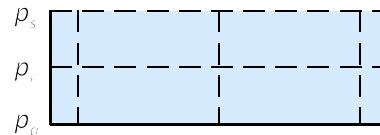
ii) with film thickness  $h_0$



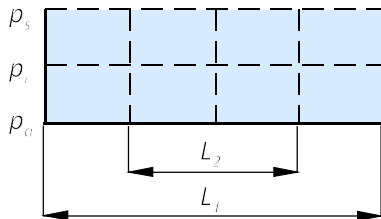
i) with film thickness  $h_{min} \rightarrow 0$



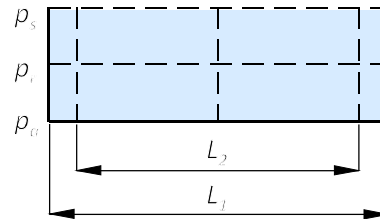
ii) with film thickness  $h_{min} \rightarrow 0$



i) with film thickness  $h_{min} = 0$



ii) with film thickness  $h_{min} = 0$



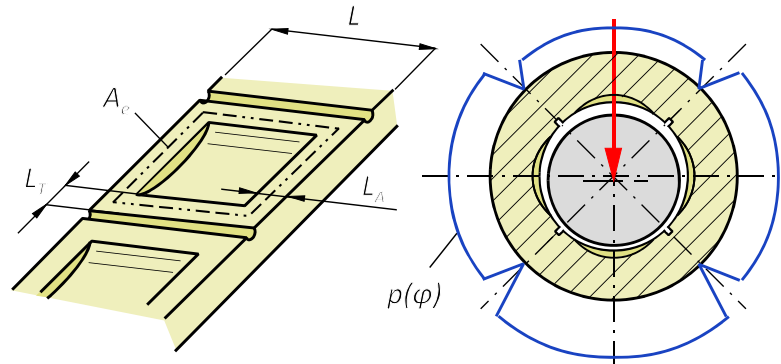
- b) What is the nominal load  $F/L$ , expressed in  $L_1$ ,  $L_2$  and  $p_s(p_a=0)$ ?
- c) What is the flow rate  $Q/L$ , expressed in  $h_0$ ,  $p_s$ ,  $\eta$ ,  $L_1$  and  $L_2$ ?
- d) What is the required pump power  $N$  [W] for 1 m bearing length, expressed in  $h_0$ ,  $F$ ,  $\eta$ ,  $L_1$  and  $L_2$ ?
- e) What should  $L_{21} = L_2/L_1$  be when a minimal pumping power is required with  $F$ ,  $h_0$ ,  $\eta$  and  $L_1$  as input?

- f) What is the optimum for  $h_1 / h_0$  if  $L_{21} = 1/2$ ?
- g) Take for bearing type i)  $L_{21} = 1/3$  and for bearing type ii)  $L_{21} = 5/6$ . Take for both bearings the same  $B$ ,  $L_1$ ,  $F$ ,  $\eta$  and  $h_0$ . Then calculate the pressure ratio  $p_{s,i}/p_{s,ii}$ , flow ratio  $Q_i/Q_{ii}$  and the power ratio  $N_i/N_{ii}$ .
- h) What is the load increase  $F_{\max}/F$  when  $h_{\min} \rightarrow 0$ ?

### Problem 11.3: E.P. 4-pocket journal bearing with capillary restrictors

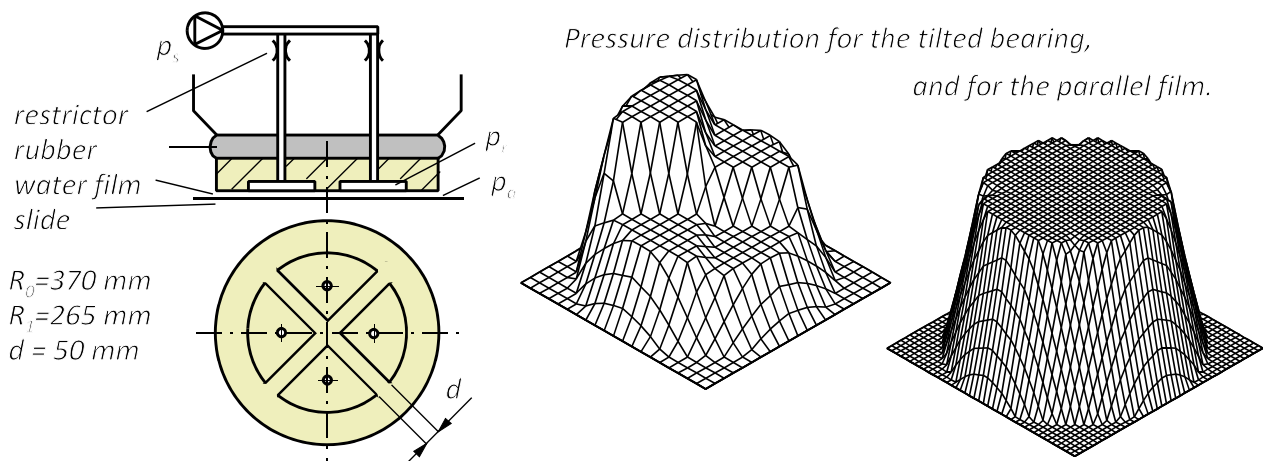
From a four-pocket journal bearing is given:  $p_s = 6 \text{ MPa}$  (60 bar gauge pressure),  $\beta_0 = 0.5$ ,  $D = 40 \text{ mm}$ ,  $L/D = 1$ ,  $L_T/L = L_A/L = 0.25$ ,  $\Delta R/R = 1/1000$ ,  $\epsilon_{\max} = 0.5$ ,  $\eta = 0.2 \text{ Pa}\cdot\text{s}$ ,  $l/d = 40$ .

- Calculate  $A_e/A$  of one of the four bearing elements.
- Calculate the bearing stiffness and load capacity.
- Calculate the flow and required pumping power.
- Calculate the dimensions of the capillary restrictors.



### Problem 11.4: E.P. 4-pocket thrust bearing with capillary restrictors

A lock gate can also be carried by a thin water film instead of wheels running on rails. The Prins Willem Alexander lock gate in the river IJ in Amsterdam are the first in the world (1995) designed in that way. The lock gates are carried on both ends by a self-aligning circular thrust bearing with four pockets. Dependent on the tilt, a pressure distribution is generated as illustrated in the figure below.



The lock gate is seven metres high, 25 metres long and three metres width. The dry mass of the lock gate amounts to approximately 180 tonnes. In the water, the weight is reduced by the air pockets in the door to 50 tonnes. Under the nominal bearing load of 250 kN on each bearing, a water film is created with a thickness of  $130 \mu\text{m}$  ( $\eta_{\text{water}} = 0.001 \text{ Pa}\cdot\text{s}$ ). The pockets are a few millimetres deep. The door can move with a velocity of  $v = 0.24 \text{ m/s}$ . Because the bearings have to carry a double overload, restrictors are

included which will reduce the feed pressure  $p_s$  with a nominal load to the pocket pressure  $p_r=0.4 p_s$  ( $p_a=0$ ). Both bearings are fed by the same pump.

- a) Give an approximation ( $\pm 10\%$ ) for the required pumping power. (Assume a linear pressure distribution and calculate the required pocket pressure, supply pressure, the flow and finally the pumping power).
- b) Calculate the friction coefficient by viscous shearing in the thin water film?
- c) During tests with a prototype hydrostatic thrust bearing, the friction coefficient proved to be  $\mu=0.001$ . This higher value is a result of the in practice incomplete separation of the bearing surfaces. If the friction coefficient in contact equals  $\mu_{BL}=0.1$  and the friction coefficient with a full film is negligible compared to this value, what part of the load is then transferred by contact?
- d) Consider a pump with a supply pressure  $p_s=2.4$  MPa (24 bar) when full restricted ( $Q=0$ ). If there is a pump pressure of 2 MPa needed to carry 250 kN, what load can be carried when i) the bearing surfaces are narrowly still separated ( $h \rightarrow 0$ ), ii) the bearing surfaces are in contact and the pressure only acts in the pocket areas?
- e) Someone proposes to apply one restrictor in the central feed rather than the separate restrictors connected to the individual pockets. The advantage would be that only one restrictor is needed for each bearing. What would be the consequence?



## Solutions

## 11.1) Linear axis with hydrostatic bearings

$$a) \quad \frac{A_e}{A} = \frac{(L_1 - L_2)(L_0 - L_2)}{L_0 L_1} = 0.53$$

$$b) \quad \frac{Q}{L} = \frac{h_0^3}{12\eta} \frac{\beta(p_s - p_a)}{L_2} \quad L = (L_0 - L_2 + L_1 - L_2)2 \quad 3600 Q = 0.162 \text{ litre/hr}$$

$$c) \quad Q = \frac{\pi d^4}{128\eta} \frac{p_s - p_r}{l_r} \quad l_r = \frac{\pi d^4}{128\eta} \frac{p_s - p_r}{Q} = 3.7 \text{ mm}$$

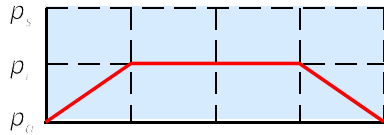
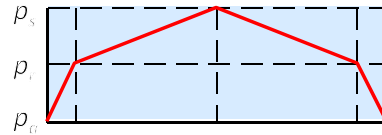
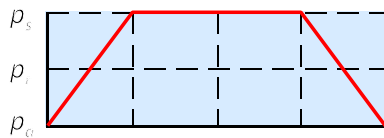
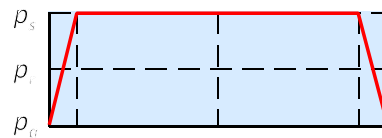
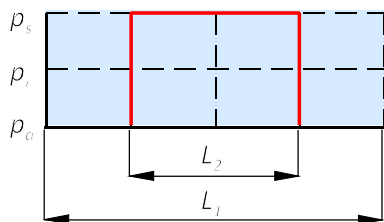
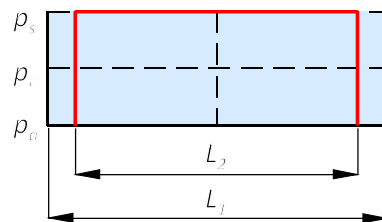
$$\text{Re} = \frac{\rho v d}{\eta} \quad v = \frac{Q}{(\pi/4)d^2} \quad \text{Re} = 2$$

$$d) \quad \left. \begin{aligned} \tau &= \frac{F_f}{A} \quad \tau = \eta \frac{du}{dz} \quad F_f = \eta v \left( \frac{A_{\text{land}}}{h_0} + \frac{A_{\text{pocket}}}{h_1} \right) \\ A_{\text{pocket}} &= (L_0 - 2L_2)(L_1 - 2L_2) \quad A_{\text{land}} = L_0 L_1 - A_{\text{pocket}} \end{aligned} \right\} F_f = 1.21 \text{ N}$$

$$e) \quad N = (p_s - p_a)Q = 0.18 \text{ W} = 0.18 \text{ J/s} \quad N = Q \rho c dT \quad dT = \frac{N}{Q \rho c} = 2.1 \text{ K}$$

## 11.2) E.P. bearing with shallow pocket

a)

i) with film thickness  $h_0$ ii) with film thickness  $h_0$ i) with film thickness  $h_{\text{min}} \rightarrow 0$ ii) with film thickness  $h_{\text{min}} \rightarrow 0$ i) with film thickness  $h_{\text{min}} = 0$ ii) with film thickness  $h_{\text{min}} = 0$ 

$$b) \quad i) \quad \frac{F}{L} = \frac{L_1 + L_2}{4} p_s \quad ii) \quad \frac{F}{L} = \frac{L_1 + 3L_2}{4} p_s$$

c) i)  $\frac{Q}{L} = \frac{h_0^3 p_s}{6\eta(L_1 - L_2)}$  ii)  $\frac{Q}{L} = \frac{h_0^3 p_s}{6\eta(L_1 - L_2)}$

d) i)  $N = \frac{h_0^3 (4F)^2}{6\eta L L_1^3 (1 - L_{21})(1 + L_{21})^2}$  ii)  $N = \frac{h_0^3 (4F)^2}{6\eta L L_1^3 (1 - L_{21})(1 + 2L_{21})^2}$

e)  $\frac{dN}{d(L_{21})} = 0$  i)  $L_{21} = 1/3$  ii)  $L_{21} = 1/2$

f)  $\left(\frac{h_0}{h_1}\right)^3 = \frac{1}{L_{21}} - 1, h_1/h_0 = 1$  with zero stiffness! This means that  $L_{21} > 1/2$  is required.

g)  $\frac{p_{s,i}}{p_{s,ii}} = \frac{(1 + 2L_{21})_{ii}}{(1 + L_{21})_i} = 2$   $\frac{Q_i}{Q_{ii}} = \frac{(1 - L_{21})_{ii}}{(1 - L_{21})_i} = \frac{1}{2}$   $\frac{N_i}{N_{ii}} = \frac{p_{s,i} Q_i}{p_{s,ii} Q_{ii}} = 1$

h)  $\left(\frac{F_{\max.}}{F}\right)_i = 2$   $\left(\frac{F_{\max.}}{F}\right)_{ii} = \frac{11}{8}$

i) i)  $\mu = \frac{4\eta U^{1-L_{21}}}{h_0 p_s^{1+L_{21}}}$

### 11.3) E.P. journal bearing

a)  $A_e = (L - L_A)(\pi D/4 - L_T), A = LD, 0.9A_e/A = 0.36$

where 0.9 is the correction for a journal bearing (page 372).

b)  $S' = \frac{Sh_0}{A(p_s - p_a)} = 6 \frac{A_e}{A} \beta_0 (1 - \beta_0) = 0.54, S = S' A p_s / h_0 = 256 \cdot 10^6 \text{ N/m}, F = S \epsilon \Delta R = 2.56 \text{ kN}$

from table 11.1 page 371, where  $h_0 = \Delta R$  and  $A_e/A$  is corrected by the factor 0.9

c)  $Q = 4 \frac{h^3}{12\eta} \frac{\beta p_s}{0.25L} ((L - 0.25L)^2 + (\frac{\pi D}{4} - 0.25L)^2), N = Q p_s$

d)  $\frac{Q}{4} = \frac{\pi d^4 (1 - \beta) p_s}{128\eta l}, l/d = 40; d = xx \text{ mm}, l = xx \text{ mm}$

### 11.4) E.P. thrust bearings

a)  $F = \pi R_1^2 p_r + \pi (R_0^2 - R_1^2) \frac{p_r}{2} = \frac{\pi}{2} (R_0^2 + R_1^2) p_r, p_r = 7.7 \text{ bar}, p_r = 0.4 p_s, p_s = 19.3 \text{ bar}$

$$Q = \frac{h^3}{12\eta} \frac{p_r}{R_0 - R_1} 2\pi \frac{R_0 + R_1}{2}, Q = 9.6 \text{ m}^3/\text{h} \quad (\text{one bearing})$$

$$N = 2 Q p_s, N = 10 \text{ kW} \quad (\text{two bearings})$$

b)  $F_f = \tau A = \eta \frac{U}{h} (\pi (R_0^2 - R_1^2) + dl) \quad l = 4(R_1 - d) + d, \mu = 2 \cdot 10^{-6}$

c)  $\mu = \alpha \mu_{\text{mech.}} + (1 - \alpha) \mu_{\text{hydr.}} \quad \alpha \approx 0.01$

d) i)  $F = \frac{250}{0.4} \frac{24}{20} = 750 \text{ kN}$       ii)  $F = p_s (\pi R_1^2 - d l) \approx 410 \text{ kN}$

- e) With this method, the self aligning capability of the bearing would be lost because the pressure in all four pockets would become independent of the bearing tilt.

## Chapter 12 - Problems with Solutions

## Problem 12.1: Circular EP airbearing pad with pressure distribution groove

Advanced high precision tooling machines are equipped with hydrostatic bearings or air bearings (main spindle, slide-ways), because of the low friction, high stiffness and smooth operation. There are some standard air bearings commercially available, as of the shelf products. But, most hydrostatic bearings and air bearings are custom designed and manufactured. Although standard bearings are made in larger series, they are still quite expensive. With some design expertise these bearings can be designed and manufactured in house for relative low cost. Approximate design solutions of the bearing properties will suffice for most applications. First step in a design process is to calculate the required bearing size.

Consider the externally pressurized (EP-) air bearing with a pressure distribution groove over the surface as shown in the figure below.

Supply pressure  $p_s = 5 \text{ bar}$  (0.5 MPa)

Ambient pressure  $p_a = 1 \text{ bar}$

Recess pressure  $p_r = 3 \text{ bar}$

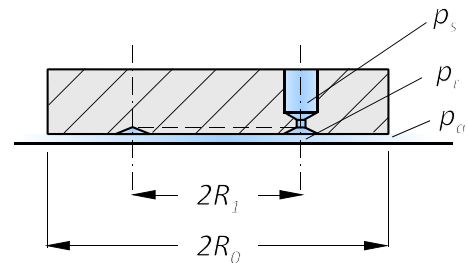
Bearing Radius  $R_0 = 25 \text{ mm}$

Groove radius  $R_1 = 20 \text{ mm}$

Groove width 0.5 mm

Groove depth  $h_2 = 0.2 \text{ mm}$

Flying height  $h_0 = 5 \text{ }\mu\text{m}$



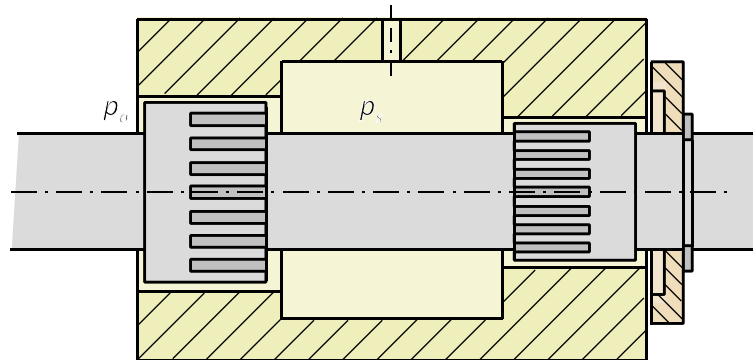
The groove depth is large compared to the flying height (film thickness) which results in a homogenous pressure distribution in the circumference of the bearing. For approximate solutions the pressure distribution is approached by a linear pressure drop from  $p_r$  to  $p_a$ .

Consider the gas properties  $R=287 \text{ m}^2/(\text{s}^2\text{K})$ ,  $\eta=18 \cdot 10^{-6} \text{ Pa s}$ ,  $\kappa=1.4$ ,  $\rho_0=1.2 \text{ kg/m}^3$  and  $C_D=0.7$

- Calculate the approximate value of the load capacity  $F$  [N] of this bearing.
- Calculate the mass flow  $M$  [ $10^{-6} \text{ kg/s}$ ].
- Calculate the diameter of the simple orifice when only one orifice is connected to the pressure distribution groove.

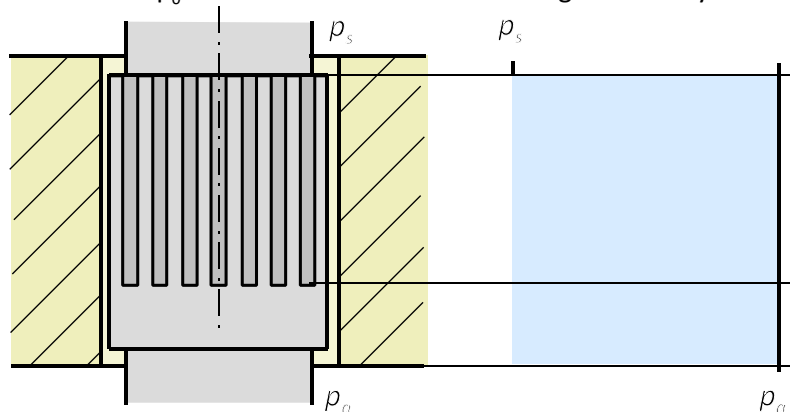
**Problem 12.2: Design of a partially grooved aerostatic journal bearing**

The bearing configuration shown below has good self aligning capability as a result of the relatively large distance between the bearings. Both bearings may be designed with different diameters or lengths, which may be profitable with eccentric loading. By applying different diameters the feeding pressure generates an axial force so that one thrust ring suffices for axial positioning. The ring will be separated from the housing by a “constant flow supply” which comes from the leakage of the partly-grooved journal bearing connected in series with the thrust ring.



The design shown above is successfully applied in a water lubricated cleaning device with supply pressures over 10MPa (100 bar) but is also applicable for gas lubrication.

a) Calculate the pressure factor  $\beta_0$  that will occur when the bearings are fed by water and air respectively.



b) Sketch the pressure distribution which may occur if the bearing is fed by water and air respectively, in the figure below.

## Solutions

## 12.1) Circular EP- air bearing pad

a) Approximate solution

$$\frac{A_E}{A} = \frac{\pi \left( \frac{R_0 + R_1}{2} \right)^2}{\pi R_0^2} = 0.81 \quad \beta = \frac{p_r - p_a}{p_s - p_a} = 0.5 \quad F' = \beta \frac{A_e}{A} = 0.405 \quad F = F' A (p_s - p_a) = 318 \text{ N}$$

The exact solution of the load capacity would result in 332 N

b)

$$M = \frac{\pi h_0^3}{12 \ln(R_0/R_1)} \frac{p_r^2 - p_a^2}{\eta R T} = 7.75 \cdot 10^{-6} \text{ kg/s}$$

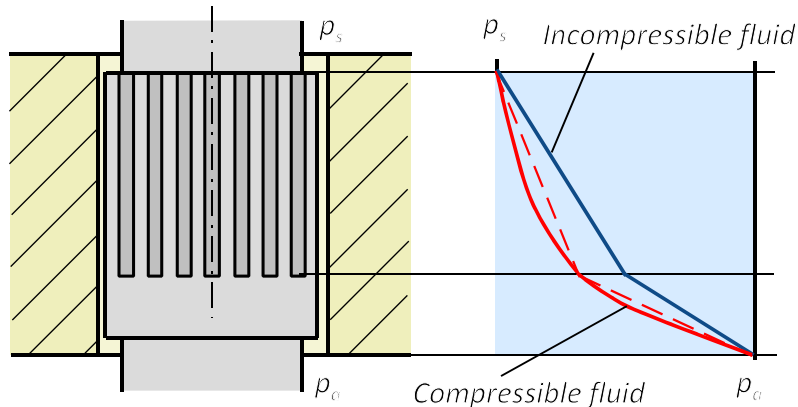
c)  $d_{\text{orifice}} = 0.25 \text{ mm}$ 

## 12.2) Partially grooved aerostatic journal bearing

a) The formula to calculate the  $\beta$ -value for the incompressible fluid is given by (eq. 11.57 page 375). Substitution of  $L'=0.9$ ,  $B'=0.1$  and  $n=4$  gives  $\beta=0.448$ . With a parallel film and incompressible fluid the pressure linearly drops from  $p_s$  to  $p_r$  and from  $p_r$  to  $p_a$ .

The formula to calculate the  $\beta$ -value for the compressible fluid is derived in the same way as described for the incompressible fluid (eq. 12.10 page 389). Working out this equation finally results in  $\beta(L', B', n, p_s/p_a)$ . Substitution of  $L'=0.9$ ,  $B'=0.1$ ,  $n=4$  and  $p_s/p_a=6$  gives  $\beta=0.681$ . For higher supply pressures,  $\beta$  approaches the asymptotic value 0.669. With a parallel film and compressible fluid the pressure distribution is convex as sketched for the stepped bearing in Figure ?.

b)



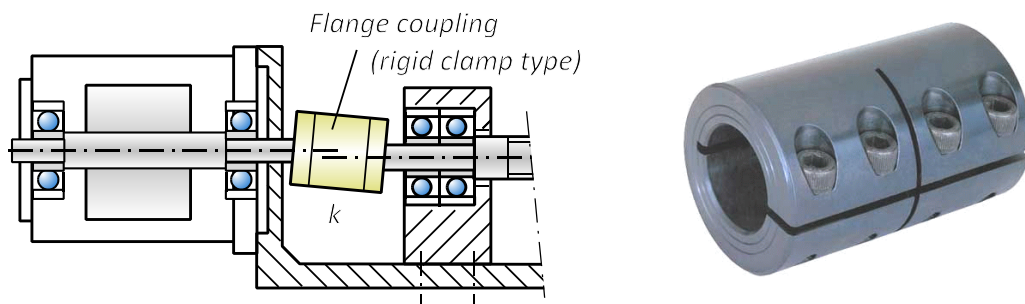
With helically shaped grooves the bearing function remains and the piston becomes driven to rotate. In the other way around, helical grooves and an externally driven rotor will provide a viscous pump. It is evident that designing e.p. air bearings and hydrostatic bearings is a creative job.

## Chapter 13 - Problems with Solutions

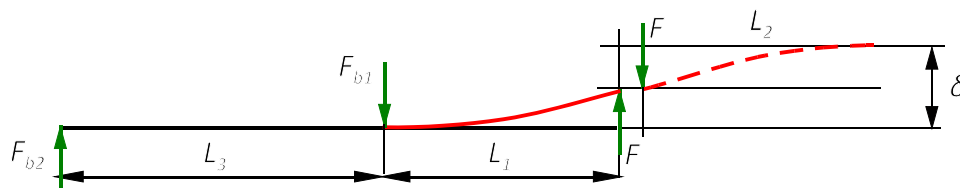
## Problem 13.1: Shaft bending by lateral misalignment

There are two types of misalignment: parallel and angular misalignment. With parallel misalignment, the center lines of both shafts are parallel but they are offset. With angular misalignment, the shafts are at an angle to each other.

When a driver like an electric motor is coupled to a screw or any other piece of equipment, it is essential that the shafts are aligned. Any misalignment between the two results in cyclic bending of the shafts and increased bearing load. This may result in premature breakdown of the equipment by fatigue fracture of the shaft or bearing failure.



Calculate the bearing load and shaft bending stress as a function of the lateral misalignment  $\delta$ .



Consider the lateral misalignment that needs to be compensated by elastic deformation of the shafts  $\delta=0.2$  mm. The more flexible part of the shaft of the electric motor and that of the spindle of length  $L_1=L_2=60$  mm and diameter  $d=12$  mm. The distance between the motor bearings is  $L_3=80$  mm.

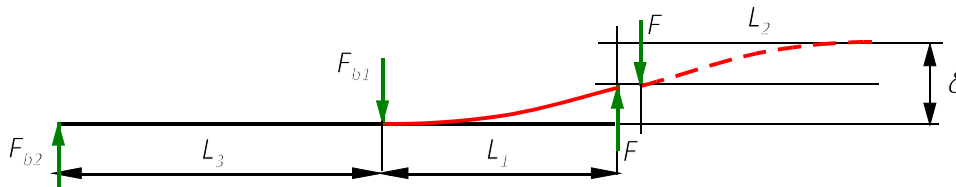
## Problem 13.2: Resonance frequency of a leaf spring guiding

The leaf springs of the linear guide described in Case 13.1 page 411 are made accidentally of a material thickness of 0.3 mm. What consequences does this have for the maximum amplitude of motion?

## Solutions

## 13.1)

The formula for lateral stiffness can be derived from the deflection mode shown in Figure ?b. Substitution of  $v(L) = \delta/2$  in (?) results in  $F = 300$  N. The moment of bending in the shoulder of the shaft becomes  $M = FL = 17.8$  Nm. This results in a bending stress of  $\sigma = 105$  MPa. The stress concentration locally will result in a much higher local stress.



The bearing load becomes  $F_{b1} = FL_1 / (L_1 + L_3) = 127$  N,  $F_{b2} = F_{b1} - F = 27$  N

## 13.2)

The stiffness will increase by a factor  $(0.3/0.25)^3$ , the maximum bending moment will increase by a factor  $(0.3/0.25)^2$ , the amplitude of motion will increase by a factor  $(0.3/0.25)^2 / (0.3/0.25)^3 = (0.25/0.3) = 5/6$ . This is a reduction of 17%. The resonant frequency will increase by a factor  $(0.3/0.25)^{3/2}$ .



## Chapter 14 - Problems with Solutions

## Problem 14.1: Self locking of metric thread and nut

Flange nuts are more resistant to vibration loosening. The wider effective bearing area result with the same friction force in a larger torque needed for self loosening. Sometimes the bearing surface is serrated to provide some extra locking action. Various alternative locking mechanisms may be employed; adhesives, safety pins, nylon inserts. Lug nuts, tapered or spherical nuts are applied to centre the nut accurately and to reduce the tendency for the nut to loosen. Fine threads have less tendency to loosen under vibration because of their smaller helix angle and larger pitch diameter  $d_2$ .



- Calculate the minimum value of the coefficient of friction needed for a M10x1.5 thread and that of an M10x1 metric fine thread in order to ensure it is self-locking.
- Calculate to what extent the torque  $M_G$ , resulting from the tread friction, will increase when unscrewing the metric fine thread.

## Problem 14.2: Torque controlled tightening

Calculate the percentage of the tightening torque needed to overcome

- the friction in the threaded contact of an M10x1.5 bolt,
- the head face contact and
- the percentage needed to develop the clamping force.

Consider the coefficient of friction of  $\mu=0.15$  and the effective bearing diameter of the nut of  $d_k=1.3d$

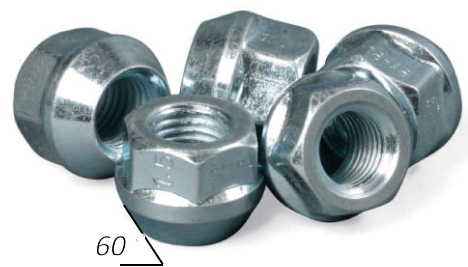


## Problem 14.3: Lug nuts and tapered wheel bolts

Lug nuts and tapered wheel bolts are designed to centre a wheel accurately on the axle and to reduce the tendency for the nut to loosen. Car manufacturers specify a proper tightening level for every fastener of the car. For M12x1.5 lug nuts with 60 degrees taper a proper torque is generally between 90 and 110 Nm (70-80 lbf-ft).

Sometimes people tighten the lug nuts until they can't turn it anymore ... then, they use their foot and body weight. With a wrench of 25 cm and a load of 80kgf this would result in  $M_A = 200$  Nm.

Calculate the initial tensile stress of the M12x1.5 stud with 60 degrees taper that would occur with the 200 Nm tightening torque. Consider a coefficient of friction of  $\mu=0.3$ ,  $d_k=1.3d$ ,  $d_2 = 11.026$  mm,  $d_3 = 10.160$  mm,  $d_0=(d_2+d_3)/2$  and  $A_t = (\pi/4)d_0^2$ .



### Problem 14.4: Torque controlled tightening

MoS<sub>2</sub> based lubricants are used in order to accommodate easy disassembly of screw joints. If the coefficient of friction in the threaded area and the head face will reduce from  $\mu = 0.3$  to  $\mu = 0.15$ , to what extent will the initial preload  $F_i$  of the screw assembly increase if the same tightening torque is applied? Calculate  $F' = F_i(\mu=0.15)/F_i(\mu=0.3)$ . Consider a head face friction of  $M_{WD} = F_i \mu d_k/2$  where  $d_k = 1.3d$ .

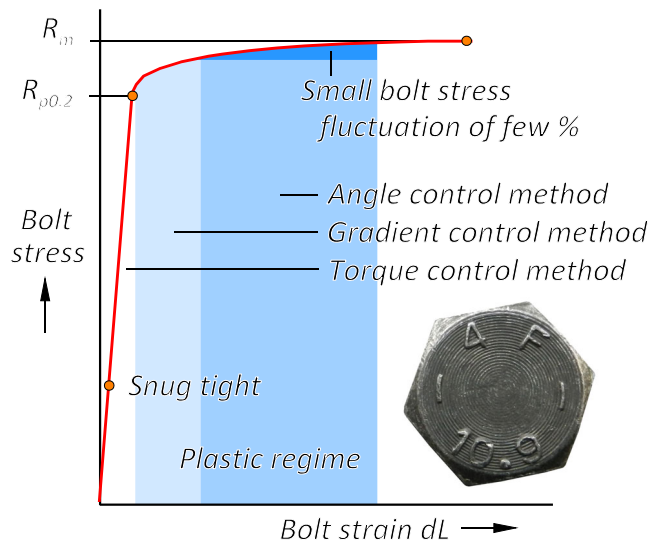
### Problem 14.5: Torque Angle controlled tightening

Torque-controlled tightening results in bolt force / clamping force deviations of  $\pm 20\%$  due to variation on the torque applied and the scatter on the coefficient of friction, despite special MoS<sub>2</sub> based lubricants are used. Therefore more advanced combined torque/angle-controlled tightening methods are applied.

Angle control tightening is a procedure in which a fastener is pre-tightened by a so called snug torque to eliminate play, and in a next step it is further tightened by giving the nut an additional controlled rotation.

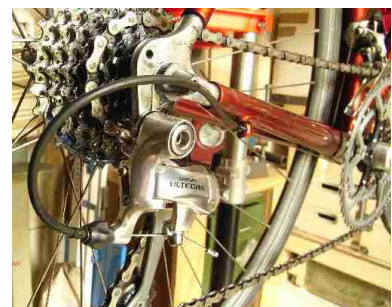
Bolts are tightened beyond their yield point by this method in order to ensure that a precise preload is achieved.

- Calculate the bolt strain when tightened to  $\sigma_i = R_{p0.2}$ .
- Calculate the bolt strain when tightened in two steps, first snug tight to 25% of the yield strength and then tightened with a 90 degrees rotation angle. Consider M10-10.9 bolt and a clamped material of  $L=5d$ .



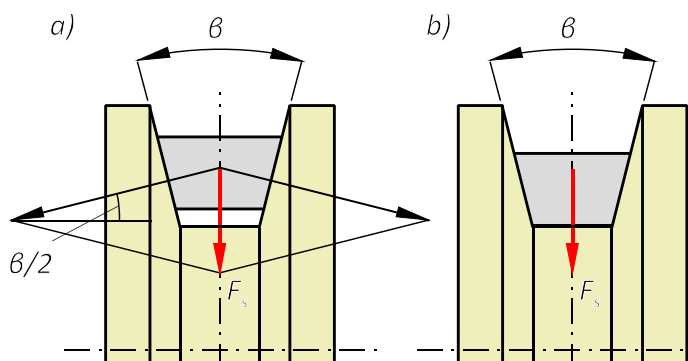
### Problem 14.6: Friction loss in Bowden cables

Calculate the percentage of loss of actuation force over the bended part of the Bowden cable shown in the figure below. Consider a coefficient of friction in a lubricated steel-steel contact of  $\mu=0.15$ .



### Problem 14.7: Wedge effect in belt pulley drives

Calculate the drive torque ratio  $T_a/T_b$ , where  $T_a$  is the torque that can be transmitted when the belt is supported on the pulley wall surfaces and  $T_b$  is the torque when the belt is supported on the groove bottom surface. Consider a pulley groove angle of  $\beta=40^\circ$ .

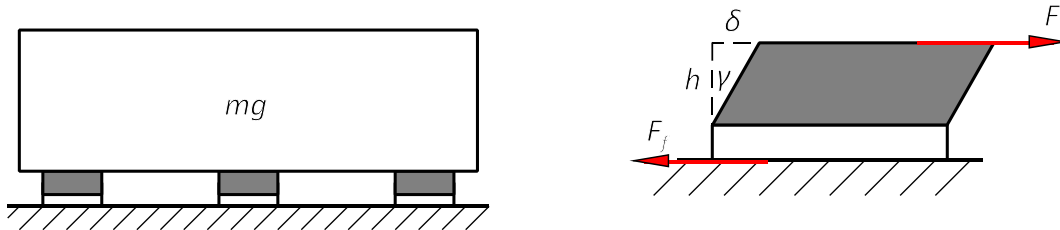


### Problem 14.8: Jamming of a piston in a cylinder

Consider a piston moving in a cylinder, as represented in Figure 5.19 page 161. The piston has a diameter  $D$  and a length  $L$ . If the load  $F_1$  acts at the piston with a maximum eccentricity of  $e=D/2$ , how long should the piston be to prevent it from jamming?

### Problem 14.9: Stick slip motion

A large concrete platform is carried on PTFE pads. The PTFE pads provide easy sliding over steel girders. The heavy load of the concrete platform is evenly distributed over the PTFE pads by using rubber blocks between the concrete platform and the PTFE pads. The concrete platform will be moved by using hydraulic cylinders. In this way an intermittent displacement of the concrete platform is to be realised.



The rubber blocks will deform elastically by the shear force, before the pads start to slide over the girders. This causes a reduction  $\delta$  of the effective stroke of movement of the hydraulic cylinders.

- Calculate the reduction of displacement  $\delta$ . Consider  $h=20$  mm, and a coefficient of friction between the PTFE pads and steel girders coated with an epoxy is  $\mu = 0.1$ , the shear modulus of the NBR rubber block  $G = 0.45$  MPa, the contact pressure over the PTFE pad  $p = 0.5$  MPa.
- Calculate the fluctuation of stick slip motion that might occur when the coefficient of friction varies between  $\mu_{\text{stat}} = 0.15$  and  $\mu_{\text{dyn}} = 0.05$ . The bearing area of the pads is  $A=0.01$  m<sup>2</sup>.

### Problem 14.10: PV-value

Consider a polymer bearing with a PV-value of  $PV = 0.2$  MPa · m/s. The shaft diameter  $d=20$  mm, the bearing width  $L=d$ , rotational speed  $n=477$  rpm.

- Calculate the load that this bearing can sustain.
- Calculate power loss in this bearing at the moment of failure, assuming a coefficient of friction  $\mu=0.2$



### Problem 14.11: Operating clearance

If a bearing bushing is press fitted in a metal housing a cumulation of machining tolerances result in a large variation of the bearing bushing inside diameter. To ensure a positive clearance under the most unfavourable conditions the minimum value of the diametral operating clearance is generally 1% of the shaft diameter ( $dR=0.05D_{\text{as}}$ ). The operating clearance is defined as the minimum clearance during operation. Effects resulting in a decrease of clearance during operation are thermal dimensional changes and for some polymers moisture-related dimensional changes.

- Calculate the decrease of the bearing clearance [%] of a PA plain bearing bushing with a shaft diameter of  $d=12$  mm and a wall thickness of  $t=3$  mm. Consider a temperature increase of the polymer of  $dT=80$  degrees and a linear expansion coefficient of PA of  $\alpha=80 \cdot 10^{-6}$  /K

---

**Problem 14.12: True or Untrue?**

1. The static coefficient of friction is generally normative in clamping joints and interference fits, explained by the stiffness of the clamped material which is in most cases smaller than the stiffness of the structure.
2. The load capacity of polymer - steel contacts subjected to sliding is limited by the PV-value, which is defined as the product of average bearing pressure and the sliding velocity.
3. The PV-value of a polymer - carbon steel contact is significantly higher than that of the same polymer - austenitic stainless steel contact.

## Solutions

- 14.1) The thread is self-locking when  $\rho' > \phi$  where  $\rho' = \tan(\mu / \cos(\beta/2))$ ,  $\beta = 60^\circ$  and  $\phi = \tan(P / \pi d_2)$ ,  $d_2 = d - 0.649519 P$ .

$$\frac{\mu}{\cos(\beta/2)} = \frac{P}{\pi d_2}$$

Substitution of  $d = 10 \text{ mm}$ ,  $P = 1.5 \text{ mm}$  gives  $\mu > 0.046$ , with  $P = 1 \text{ mm}$  then  $\mu > 0.029$ . It can be concluded that fine threads are less susceptible to vibration loosening.

The torque necessary to overcome thread friction when unscrewing is:

$$M_G = F_i \frac{d_2}{2} \tan\left(\tan^{-1}\frac{P}{\pi d_2} - \tan^{-1}\frac{\mu}{\cos(\beta/2)}\right) \quad \frac{M_G(M10 \times 1)}{M_G(M10 \times 1.5)} = 1.151$$

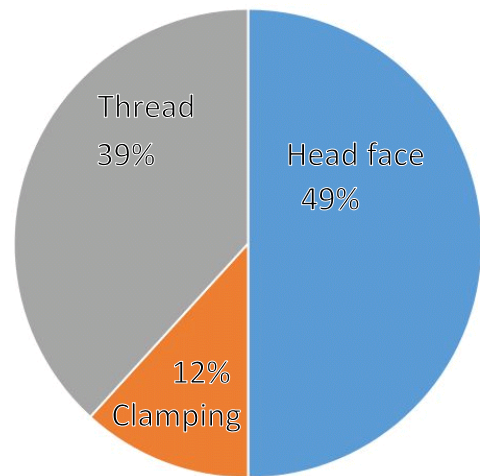
- 14.2) Total friction

$$M = F_i \frac{d_2}{2} \tan\left[\tan^{-1}\left(\frac{P}{\pi d_2}\right) + \tan^{-1}\left(\frac{\mu}{\cos(\beta/2)}\right)\right] + \mu F_i \frac{d_k}{2}$$

- Percentage head face friction:  $\frac{\mu F_i \frac{d_k}{2}}{M} = 49\%$

- Percentage thread friction:  $\frac{F_i \frac{d_2}{2} \tan\left[\tan^{-1}\left(\frac{\mu}{\cos(\beta/2)}\right)\right]}{M} = 39\%$

- Percentage clamping:  $\frac{F_i \frac{d_2}{2} \tan\left[\tan^{-1}\left(\frac{P}{\pi d_2}\right)\right]}{M} = 12\%$



14.3) 
$$F_i = \frac{M}{\frac{d_2}{2} \tan\left[\tan^{-1}\left(\frac{P}{\pi d_2}\right) + \tan^{-1}\left(\frac{\mu}{\cos(\beta/2)}\right)\right] + \mu \frac{d_k}{2} \frac{1}{\cos(\alpha)}} \quad \sigma_i = \frac{F_i}{A_t} = 397 \text{ MPa}$$

The head face friction has increased by a factor  $1/\cos(\alpha)=2$  and becomes 68% of  $M_A$ .

14.4) 
$$F_i(\mu) = \frac{M}{\frac{d_2}{2} \tan\left[\tan^{-1}\left(\frac{P}{\pi d_2}\right) + \tan^{-1}\left(\frac{\mu}{\cos(\beta/2)}\right)\right] + \mu \frac{d_k}{2}} \quad \frac{F_i(\mu=0.15)}{F_i(\mu=0.3)} = 1.89$$

If a preload of  $F_i = 0.8 F_{0.2}$  was intended with a coefficient of friction of  $\mu=0.3$ , and the same tightening torque would be applied when lubricated with the  $\text{MoS}_2$  based lubricant then the screw joint would be overstretched during tightening and will break off.

14.5) 
$$\epsilon = \frac{R_{p0.2}}{E_{st}} = \frac{0.9 \cdot 1000 \cdot 10^6}{210 \cdot 10^9} = 0.43\%$$

$$\epsilon = \frac{1}{4} \frac{R_{p0.2}}{E_{st}} + \frac{60}{360} \frac{P}{3d} = \frac{0.43\%}{4} + 0.83\% = 0.94\%$$

$$14.6) \quad \frac{T_1}{T_2} = e^{\mu\alpha} \quad \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1} = 1 - e^{-\mu\alpha} = 0.376$$

$$14.7) \quad \left. \begin{array}{l} F_{\perp} = \frac{F}{2 \sin(\beta/2)} \quad F_{f,a} = 2\mu F_{\perp} \\ F_{f,b} = \mu F \end{array} \right\} \quad \frac{F_{f,a}}{F_{f,b}} = \frac{1}{\sin(\beta/2)}$$

14.8) The system jams when  $\mu > \frac{1}{2} h/e$ . Substitution of  $h = L$  and  $e = D/2$  gives  $\mu > L/D$ . This means that with  $L/D > \mu$  the piston will not jam in the circumstances given.

$$14.9) \quad \gamma = \arctan\left(\frac{\delta}{h}\right) \quad \delta = h \tan\left(\frac{\tau}{G}\right) \quad \tau = \frac{F_f}{A} \quad F_f = \mu p A \quad \text{than} \quad \delta = h \tan\left(\frac{\mu p}{G}\right) = 2.23 \text{ mm}$$

Figure 1.26 page 30:  $\Delta F = 2(\mu_s - \mu_d)pA \quad \Delta X = \Delta F/k \quad k = \frac{F_f}{\delta} \quad \text{than} \quad \Delta X = 4.46 \text{ mm}$

14.10)

$$a) \quad F = \frac{PV}{v} A \quad A = dL \quad v = \pi d n \quad v = 0.5 \text{ m/s} \quad F = 160 \text{ N}$$

$$b) \quad P = F_f v \quad F_f = \mu F \quad P = 16 \text{ W}$$

$$14.11) \quad \Delta t = t \alpha d T = 0.019 \text{ mm} \quad \Delta d = 2 \Delta t \quad \Delta d/d = 0.32\%$$

14.12)

Untrue: 1

True: 2, 3

# Engineering Equations

---

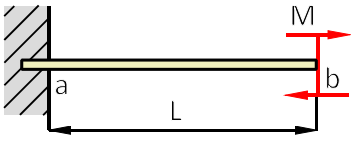
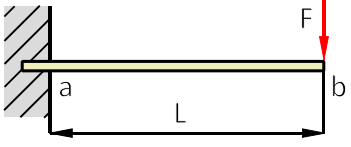
## Data Tables

Table A1: Conversion factors to SI Units

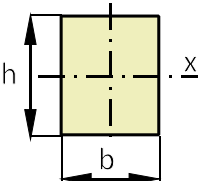
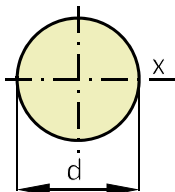
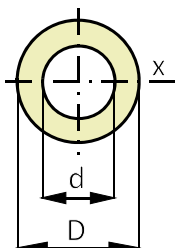
<p><b>Length:</b></p> <p>1 foot (ft) = 1200/3937 m</p> <p>1 inch (in) = 25.4E-03 m</p> <p>1 mile = (5280 feet)</p> <p><b>Mass:</b></p> <p>1 slug = 14.59 kg</p> <p>1 pound (lb) = 0.454 kg</p> <p><b>Force:</b></p> <p>1 pound (lb) = 4.448 N</p> <p>1 dyne = 1E-05 N</p> <p>1 kgf = 9.81 N</p> <p><b>Pressure:</b></p> <p>1 lb/in<sup>2</sup> = 6895 Pa</p> <p>1 bar = 1E+05 Pa</p> <p>1 psi = (1 lb/in<sup>2</sup>)</p> <p><b>PV:</b></p> <p>1 psi·fps = 2.1E03 Pa·m/s</p> <p>1 psi·fpm = 35.0E+05 Pa·m/s</p>	<p><b>Power:</b></p> <p>1 ft-lb/s = 1.356 W</p> <p>1 hp = 746 W</p> <p><b>Volume:</b></p> <p>1 gal (US) = 3.785E-03 m<sup>3</sup></p> <p>1 gal (UK) = 4.546E-03 m<sup>3</sup></p> <p>1 barrel = (42 gallon)</p> <p><b>Temperature:</b></p> <p>Celsius (°C) = (°F - 32)5/9</p> <p>Kelvin (K) = °C + 273</p> <p><b>Dynamic viscosity:</b></p> <p>1 cP = 1E-03 Pa·s</p> <p>1 poise (P) = (100 cP)</p> <p><b>Kinematic viscosity:</b></p> <p>1 cSt = 1E-06 m<sup>2</sup>/s</p> <p>1 stokes (St) = (100 cSt)</p>
---	---



**Table A2** Deflections and slopes of uniform cantilever beams

Loading	deflection	slope
	$\delta_b = \frac{ML^2}{2EI}$	$\theta = \frac{ML}{EI}$
	$\delta_b = \frac{FL^3}{3EI}$	$\theta = \frac{FL^2}{2EI}$

**Table A3** Moments of inertia  $I_x$ ,  $I_y$  and Polar moments of inertia  $I_p$ 

Cross section	Bending	Torsion
Elementary equations for uniform beams subjected to bending and torsion respectively:	$M_x = S_x \sigma_b, \quad S_x = \frac{I_x}{y_{\max}}$ $I_x = \int y^2 dA$ $\delta = \frac{ML^2}{2EI}, \quad \theta = \frac{ML}{EI}$	$T = S_t \tau_{\max}, \quad S_t = \frac{I_p}{r}$ $I_p = I_x + I_y$ $\phi = \frac{TL}{GI_p}, \quad G = \frac{E}{2(1+\nu)}$
	$I_x = \frac{1}{12} b h^3$ $S_x = \frac{1}{6} b h^2$	
	$I_x = \frac{\pi}{64} d^4$ $S_x = \frac{\pi}{32} d^3$	$I_p = \frac{\pi}{32} d^4$ $S_t = \frac{\pi}{16} d^3$
	$I_x = \frac{\pi}{64} (D^4 - d^4)$ $S_x = \frac{\pi}{32} \frac{D^4 - d^4}{D}$	$I_p = \frac{\pi}{32} (D^4 - d^4)$ $S_t = \frac{\pi}{16} \frac{D^4 - d^4}{D}$

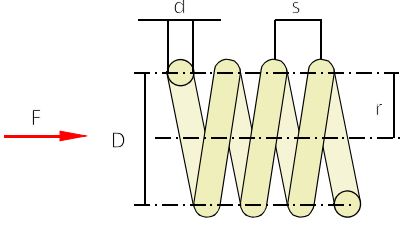
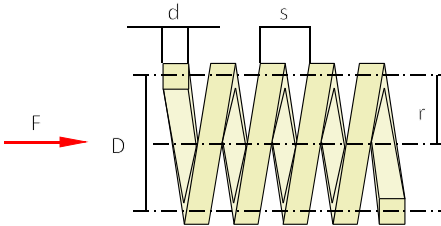
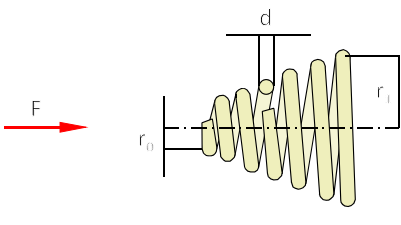
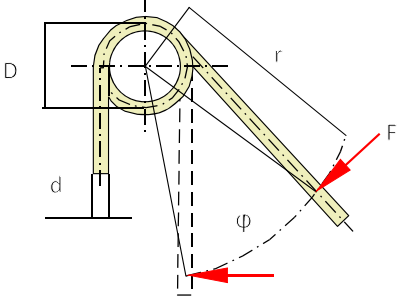
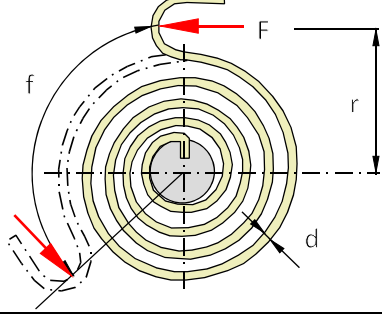
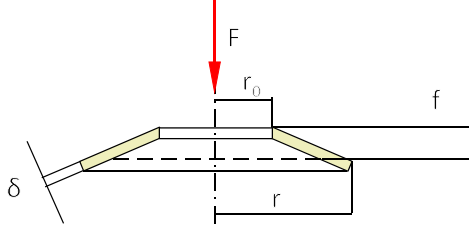
von Mises equivalent stress  $\sigma_e = \sqrt{\sigma^2 + 3(\alpha_0 \tau^2)}$

$\alpha_0 = 1$  when  $\sigma$  and  $\tau$  are both static or fully reversed,

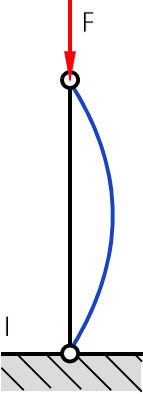
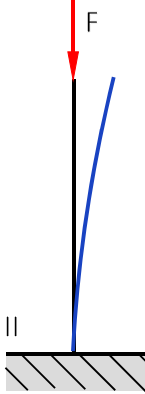
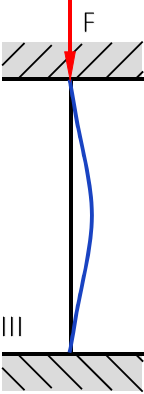
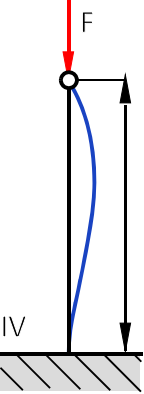
$\alpha_0 = 0.7$  when  $\sigma$  is fully reversed and  $\tau$  remains static

$\alpha_0 = 1.5$  when  $\sigma$  remains static and  $\tau$  is fully reversed

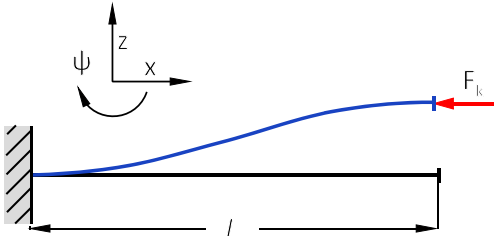
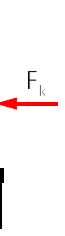
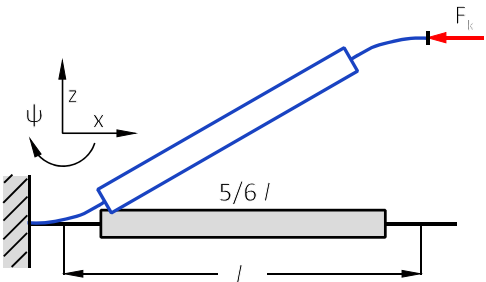
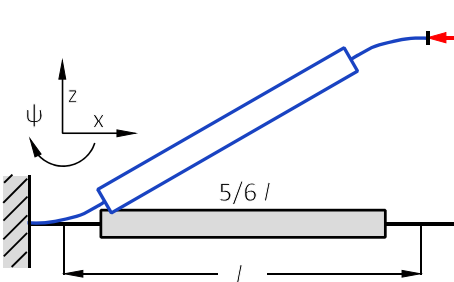
**Table A4** Approximate formulae for spring stiffness

Spring	Stiffness
	$F = \frac{\pi}{16} \frac{d^3}{r} \tau$ $f = \frac{64 n r^3 F}{d^4 G}$
	$F = \frac{d^3}{4.8 r} \tau$ $f = \frac{44.9 n r^3 F}{d^4 G}$
	$F = \frac{\pi}{16} \frac{d^3}{r_1} \tau$ $f = \frac{16 n (r_1 + r_0)(r_1^2 + r_0^2) F}{d^4 G}$
	$F = \frac{\pi}{32} \frac{d^3}{r} \sigma$ $\phi = \frac{2 D \pi n}{E d} \sigma \frac{180}{\pi}$ <p><math>\phi</math> [deg.]</p>
	$F = \frac{\pi}{32} \frac{d^3}{r} \sigma$ $f = \frac{2 r l}{E d} \sigma$ <p><math>l</math> = eff spring length</p>
	$F = \frac{\delta^2}{1 - 2/3 r_0/r} \sigma$ $f = \frac{0.65 r^2}{E \delta (1 - 2/3 r_0/r)} \sigma$

**Table A5** Buckling limit of compression loaded beams [Gero & Timoshenko, 1985]

			
$\frac{FL^2}{EI} = \pi^2$	$\frac{FL^2}{EI} = \frac{\pi^2}{4}$	$\frac{FL^2}{EI} = 4\pi^2$	$\frac{FL^2}{EI} = 2.046\pi^2$

**Table A6** Approximate design functions S-shaped beams [Koster, 1996]

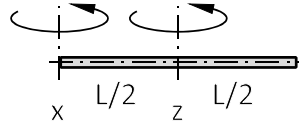
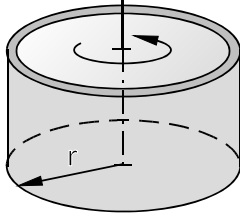
				
	leaf spring	wire spring	leaf spring	wire spring
longitudinal stiffness $C_{xx}$	$\frac{EA}{l}$	$\frac{EA}{l}$	$\frac{3EA}{l}$	$\frac{3EA}{l}$
lateral stiffness $C_{zz}$	$\frac{12EI}{l^3}$	$\frac{12EI}{l^3}$	$\frac{72EI}{5l^3}$	$\frac{72EI}{5l^3}$
bending stress $\sigma_{\psi z}$	$\frac{3Etz}{l^2}$	$\frac{3Edz}{l^2}$	$\frac{3Etz}{l^2}$	$\frac{3Edz}{l^2}$
buckling load $F_k$	$\frac{4\pi^2 EI}{l^2}$	$\frac{4\pi^2 EI}{l^2}$	$\frac{36\pi^2 EI}{l^2}$	$\frac{36\pi^2 EI}{l^2}$

<sup>1)</sup> The configuration with reinforced mid-section considered in this table shows an increase of the buckling load by a factor of nine while the lateral stiffness has increased with only 20%.

**Table A7** Moments of Inertia

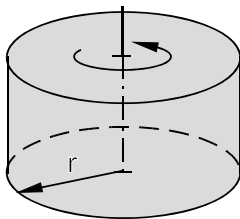
Moment of inertia:  $I = \int \rho^2 dm$  Radius of gyration:  $i = \sqrt{I/m}$

Parallel axis theorem:  $I = I_z + a^2 m$  Torque:  $T = I\alpha$   $\alpha = \dot{\omega}$

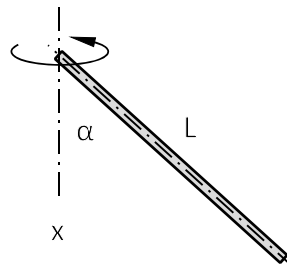


$$I_z = \frac{1}{12} mL^2$$

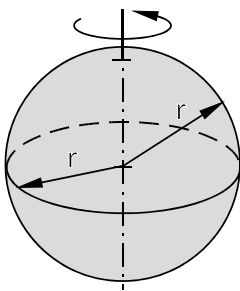
$$I_x = I_z + \left(\frac{L}{2}\right)^2 m$$



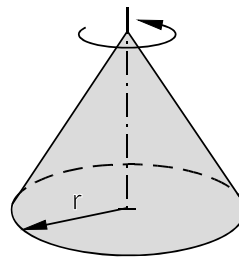
$$I = \frac{1}{2} mr^2$$



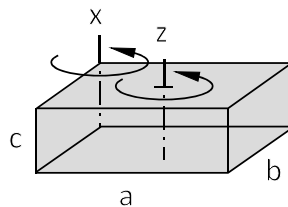
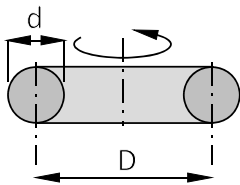
$$I_x = \frac{1}{3} m(L \sin \alpha)^2$$



$$I = \frac{2}{5} mr^2$$



$$I = \frac{3}{10} mr^2$$



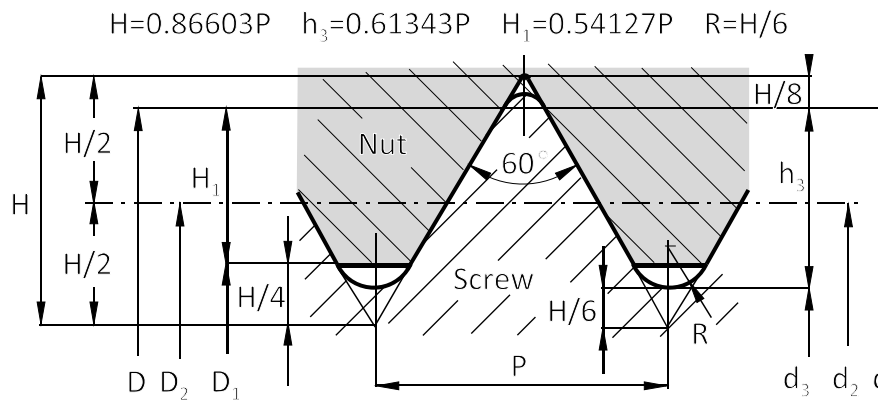
$$I_z = \frac{1}{12} m(a^2 + b^2)$$

$$I_x = \frac{1}{3} m(a^2 + b^2)$$

**Linear motion****Rotational motion**Position  $x$ Angular position  $\phi$ Velocity  $v$ Angular velocity  $\omega$ Acceleration  $a$ Angular acceleration  $\alpha$ Load  $F$  [ N ]moment  $M$  [ Nm ]mass  $m$  [ kg ]moment of inertia  $I$  [ kg m<sup>2</sup> ]Impulse  $p = m v$  [ kg m/s ] Angular Momentum  $H = I \omega$  [ kg m<sup>2</sup>/s ] $F = m a$  [ N = kg m/s<sup>2</sup> ]  $M = I \alpha$  [ Nm = kg m<sup>2</sup> rad/s<sup>2</sup> ] $W = F s$  [ N m = J ]  $W = M \phi$  [ Nm rad = J ] $E_k = \frac{1}{2} mv^2$  [ J ]  $E_k = \frac{1}{2} I \omega^2$  [ J ] $P = W/t = F s / t$  [ J/s = W ]  $P = W/t = M \omega$  [ J/s = W ]

**Table A8** ISO Tolerances for holes and shafts

## Metric screw threads ISO 724 (DIN 13 T1)



Nominal size d = D	Pitch P	root radius r	pitch diameter d <sub>2</sub> =D <sub>2</sub>	minor diameter d <sub>3</sub> D <sub>1</sub>		thread height h <sub>3</sub> H <sub>1</sub>		stress area A <sub>s</sub>	drill diam. mm
M 1	0.25	0.036	0.838	0.693	0.729	0.153	0.135	0.460	0.75
M 1.2	0.25	0.036	1.038	0.893	0.929	0.153	0.135	0.732	0.95
M 1.6	0.35	0.051	1.373	1.171	1.221	0.215	0.189	1.27	1.25
M 2	0.40	0.058	1.740	1.509	1.567	0.245	0.217	2.07	1.60
M 2.5	0.45	0.065	2.208	1.948	2.013	0.276	0.244	3.39	2.05
M 3	0.50	0.072	2.675	2.387	2.459	0.307	0.271	5.03	2.50
M 4	0.70	0.101	3.545	3.141	3.242	0.429	0.379	8.78	3.30
M 4.5	0.75	0.108	4.013	3.580	3.688	0.460	0.406	11.3	3.80
M 5	0.80	0.115	4.480	4.019	4.134	0.491	0.433	14.2	4.20
M 6	0.100	0.144	5.350	4.773	4.917	0.613	0.541	20.1	5.00
M 8	0.125	0.180	7.188	6.466	6.647	0.767	0.677	36.6	6.80
M 9	1.25	0.180	8.188	7.466	7.647	0.767	0.677	48.1	7.80
M 10	1.50	0.217	9.026	8.160	8.376	0.920	0.812	58.0	8.50
M 11	1.50	0.217	10.026	9.160	9.376	0.920	0.812	72.3	9.50
M 12	1.75	0.253	10.863	9.853	10.106	1.074	0.947	84.3	10.20
M 14	2.00	0.289	12.701	11.546	11.835	1.227	1.083	115	12.00
M 16	2.00	0.289	14.701	13.546	13.835	1.227	1.083	157	14.00
M 18	2.50	0.361	16.376	14.933	15.394	1.534	1.353	193	15.50
M 20	2.50	0.361	18.376	16.933	17.294	1.534	1.353	245	17.50
M 22	2.50	0.361	20.376	18.933	19.294	1.534	1.353	303	19.50
M 24	3.00	0.433	22.051	20.319	20.752	1.840	1.624	353	21.00
M 27	3.00	0.433	25.051	23.319	23.752	1.840	1.624	459	24.00
M 30	3.50	0.505	27.727	25.706	26.211	2.147	1.894	561	26.50
M 33	3.50	0.505	30.727	28.706	29.211	2.147	1.894	694	29.50
M 36	4.00	0.577	33.402	31.093	31.670	2.454	2.165	817	32.00
M 42	4.50	0.650	39.077	36.479	37.129	2.760	2.436	1121	37.50
M 48	5.00	0.722	44.752	41.866	42.857	3.067	2.706	1473	43.00
M 56	5.50	0.794	52.428	49.252	50.046	3.374	2.977	2030	50.50

Table A9 ISO Tolerances for holes and shafts

Example fits using ISO hole basis													
	clearance fit				transition fit				interference fit				
H6	h5				j6, k6				n5, r5				
H7	f7, g6, h6				k6, m6, n6				r6, s6				
H8	d9, e8, f8, h9								s8, u8, x8				

ISO Tolerances for holes (ISO 286-2)													
Nominal hole sizes (mm)													
>	3	6	10	18	30	40	50	65	80	100	120	140	160
≤	6	10	18	30	40	50	65	80	100	120	140	160	180
micrometer													
H6	+8 0	+9 0	+11 0	+13 0	+16 0	+19 0	+22 0	+25 0					
H7	+12 0	+15 0	+18 0	+21 0	+25 0	+30 0	+35 0	+40 0					
H8	+18 0	+22 0	+27 0	+33 0	+39 0	+46 0	+54 0	+63 0					

ISO Tolerances for shafts (ISO 286-2)													
Nominal shaft sizes (mm)													
>	3	6	10	18	30	40	50	65	80	100	120	140	160
≤	6	10	18	30	40	50	65	80	100	120	140	160	180
micrometer													
f6	-10 -18	-13 -22	-16 -27	-20 -33	-25 -41	-30 -49	-36 -58	-43 -68					
f7	-10 -22	-13 -28	-16 -34	-20 -41	-25 -50	-30 -60	-36 -71	-43 -83					
g6	-4 -12	-5 -14	-6 -17	-7 -20	-9 -25	-10 -29	-12 -34	-14 -39					
g7	-4 -16	-5 -20	-6 -24	-7 -28	-9 -34	-10 -40	-12 -47	-14 -54					
h5	-0 -5	-0 -6	-0 -8	-0 -9	-0 -11	-0 -13	-0 -15	-0 -18					
h6	-0 -8	-0 -9	-0 -11	-0 -13	-0 -16	-0 -19	-0 -22	-0 -25					
h7	-0 -12	-0 -15	-0 -18	-0 -21	-0 -25	-0 -30	-0 -35	-0 -40					
h9	-0 -30	-0 -36	-0 -43	-0 -52	-0 -62	-0 -74	-0 -87	-0 -100					
j6	+6 -2	+7 -2	+8 -3	+9 -4	+11 -5	+12 -7	+13 -9	+14 -11					
k6	+9 +1	+10 +1	+12 +1	+15 +2	+18 +2	+21 +2	+25 +3	+28 +3					
m6	+12 +4	+15 +6	+18 +7	+21 +8	+25 +9	+30 +11	+35 +13	+40 +15					
n5	+13 +8	+16 +10	+20 +12	+24 +15	+28 +17	+33 +20	+38 +23	+45 +27					
n6	+16 +8	+19 +10	+23 +12	+28 +15	+33 +17	+39 +20	+45 +23	+52 +27					
p6	+20 +12	+24 +15	+29 +18	+35 +22	+42 +26	+51 +32	+59 +37	+68 +43					
r6	+23 +15	+28 +19	+34 +23	+41 +28	+50 +34	+60 +41	+62 +43	+73 +51	+76 +54	+88 +63	+90 +65	+93 +68	